

Refraction Correction Equations and Tables

(NASA-TM-80886) REFRACTION CORRECTION
EQUATIONS AND TABLES (NASA) 287 p

N80-70980

Unclassified
00/32 42203

Mission Planning and Analysis Division
October 1979



National Aeronautics and
Space Administration

Lyndon B. Johnson Space Center
Houston, Texas



79FM42

79-FM-42

JSC-16234

SHUTTLE PROGRAM

REFRACTION CORRECTION EQUATIONS AND TABLES

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1.0 INTRODUCTION

Measurements of range and elevation angle are distorted by the Earth's atmosphere, as shown in figure 1-1.

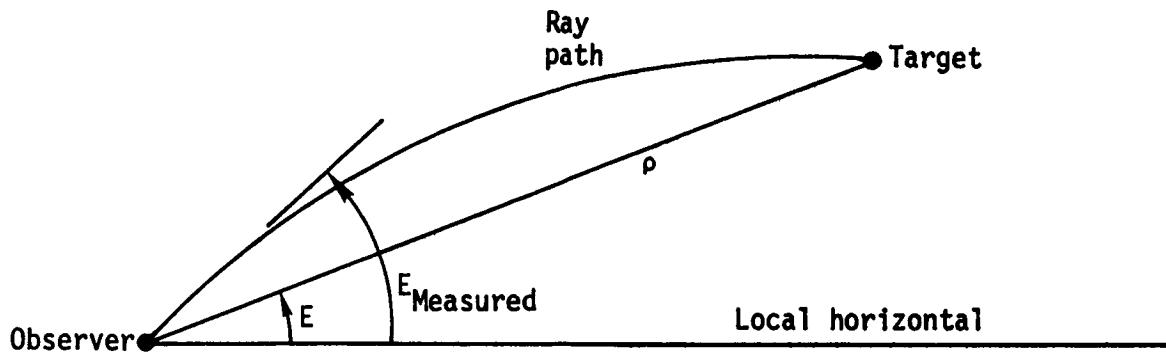


Figure 1-1.- Refraction effects.

In this document, the differences between the measured and geometric values of these quantities are investigated.

Four different methods of obtaining the refraction corrections are discussed.

- a. To ray trace, the oldest method. The atmosphere is broken into layers and is ray traced using the spherical form of Snell's law. The inputs are measured elevation angle and generally, final altitude.
- b. To solve a set of integral equations whose inputs are measured elevation angle and final altitude. Gaussian quadrature is used to solve the equations.
- c. To solve a set of three nonlinear differential equations whose inputs are measured elevation angle and measured range. This method gives the greatest accuracy with the least amount of effort.
- d. To use approximations based on a spherical slab atmosphere. The inputs are geometric elevation angle and geometric range. The equations are relatively simple and may be rapidly evaluated, thus lending themselves for use in real-time computers.

Also included in this report are extensive tables of very precise refraction corrections that may be used to evaluate various approximations.

2.0 THE EXPONENTIAL ATMOSPHERE

Let

R_o = radius of Earth at observer = 6 378 165 meters (value is not critical)

h = altitude above R_o

$$h = \sqrt{R_o^2 + \rho^2 + 2R_o \rho \sin E} - R_o \quad (2.1)$$

n = index of refraction at h

N = $n-1$, modulus of refraction

N_o = value of N at R_o , supplied by meteorologists (on the Earth)

$$0.000255 \leq N_o \leq 0.000395$$

H_S = atmospheric scale height constant supplied by meteorologists

The exponential atmospheric model is defined by

$$N = N_o \exp(-h/H_S) \quad (2.2)$$

For processing radar tracking data, Goddard Spaceflight Center and Johnson Space Center use the following empirical relation for determining H_S .

$$H_S = \frac{1000}{\ln \frac{N_o}{N_o - 7.32 \cdot 10^{-6} \exp(5577 N_o)}} \text{ meters} \quad (2.3)$$

The source of equation (2.3) is reference 1. Substituting this equation into equation (2.2) and setting $h = 1000$ meters yields

$$N_{1000} = N_o - 7.32 \cdot 10^{-6} \exp(5577 N_o) \quad (2.4)$$

That is, H_S is based on the modulus of refraction at 1000 meters above the observer. Thus, this value of H_S may not be best for high altitude satellite tracking. However, equation (2.3) was used in obtaining the results shown in this report.

Typical values of H_S are shown as follows:

$$N_0 = 0.000255 \quad 0.000290 \quad 0.000325 \quad 0.000360 \quad 0.000395$$

$$H_S = 7829 \quad 7350 \quad 6735 \quad 6091 \quad 5446 \text{ meters}$$

For error analysis purposes, H_S given by equation (2.3) may be approximated by a polynomial in N_0 . A first-order approximation is

$$H_S = 12\ 470 - 17.715 \cdot 10^6 N_0 \quad (2.5)$$

with a 1σ error of 22 meters. A third-order approximation is

$$H_S = 6402.05 + 35.05462 \cdot 10^6 N_0 - 0.1503623 \cdot 10^{12} N_0^2 + 1.405094 \cdot 10^{14} N_0^3 \quad (2.6)$$

with a 1σ error of 0.74 meters.

At Patrick Air Force Base a different method is used to determine H_S (ref. 2).

$$H_S = \frac{100\ 000}{\ln(N_0/\bar{N}_{100\ 000})} \text{ feet} \quad (2.7)$$

where $\bar{N}_{100\ 000}$ is the mean yearly value of N at 100 000 feet above the tracking station. At Cape Canaveral,

$$\bar{N}_{100\ 000} = 3.36 \cdot 10^{-6}$$

Using a summertime Cape value of $N_0 = 0.000395$ gives $H_S = 6397$ meters, a significant difference from the 5446 meters obtained previously.

3.0 THE SPHERICAL FORM OF SNELL'S LAW

A ray traveling from a medium with an index of refraction of n_i to a medium with an index of refraction of n_{i+1} is bent according to Snell's law (fig. 3-1).

$$n_{i+1} \sin \phi_{i+1} = n_i \sin \phi_i \quad (3.1)$$

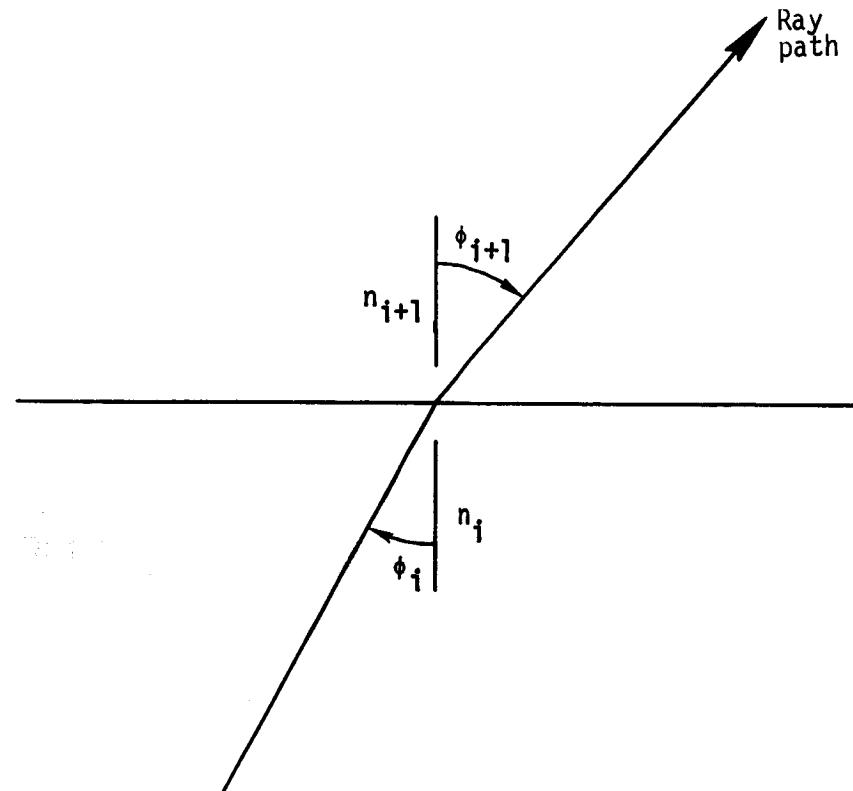


Figure 3-1.- Snell's law.

Snell's law will now be applied to concentric spherical surfaces (fig. 3-2).

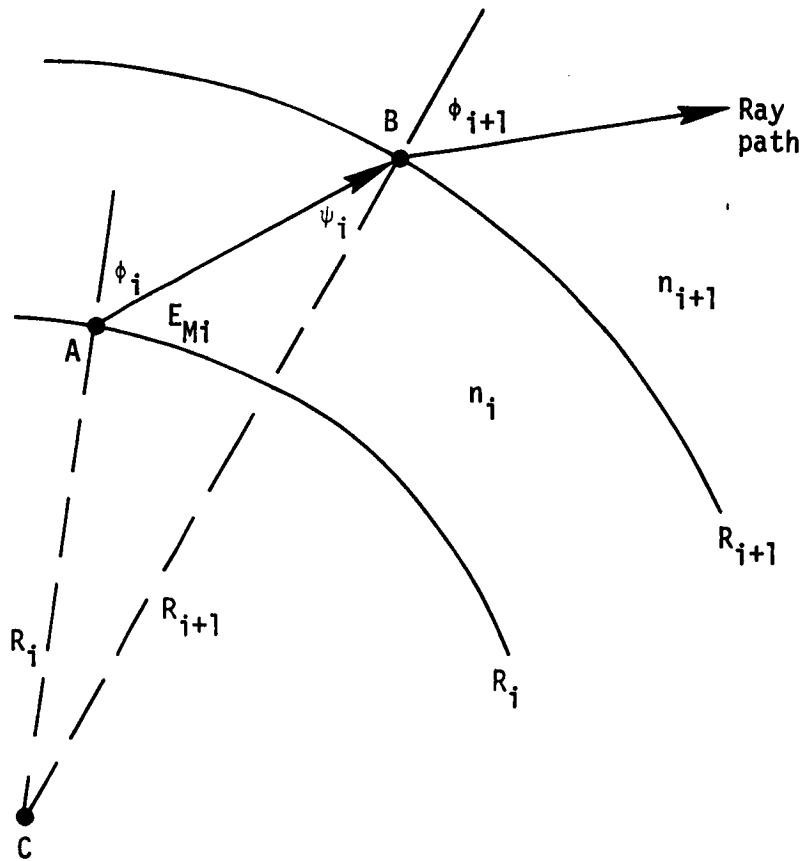


Figure 3-2.- Ray path across spherical surfaces.

From the ABC triangle in figure 3-2 and from the law of sines

$$\frac{R_i}{\sin \psi_i} = \frac{R_{i+1}}{\sin(180-\phi_i)} = \frac{R_{i+1}}{\sin \phi_i}$$

or

$$R_{i+1} \sin \psi_i = R_i \sin \phi_i$$

But from Snell's law of refraction

$$\sin \psi_i = \frac{n_{i+1}}{n_i} \sin \phi_{i+1} \quad (3.2)$$

Thus

$$R_{i+1} \frac{n_{i+1}}{n_i} \sin \phi_{i+1} = R_i \sin \phi_i$$

or

$$n_{i+1} R_{i+1} \sin \phi_{i+1} = n_i R_i \sin \phi_i \quad (3.3)$$

And in the next layer it is easily shown that

$$n_{i+2} R_{i+2} \sin \phi_{i+2} = n_i R_i \sin \phi_i$$

Or, in general,

$$nR \sin \phi = n_i R_i \sin \phi_i \quad (3.4)$$

Elevation angle E_M is defined by

$$E_M = 90 - \phi$$

Substituting equation (3.4) gives the spherical form of Snell's law:

$$\boxed{! \quad nR \cos E_M = n_i R_i \cos E_{Mi} \quad !} \quad (3.5)$$

4.0 RAY TRACING EQUATIONS

The atmosphere will be divided into k layers. The j layer ($1 \leq j \leq k$) is shown in figure 4-1.

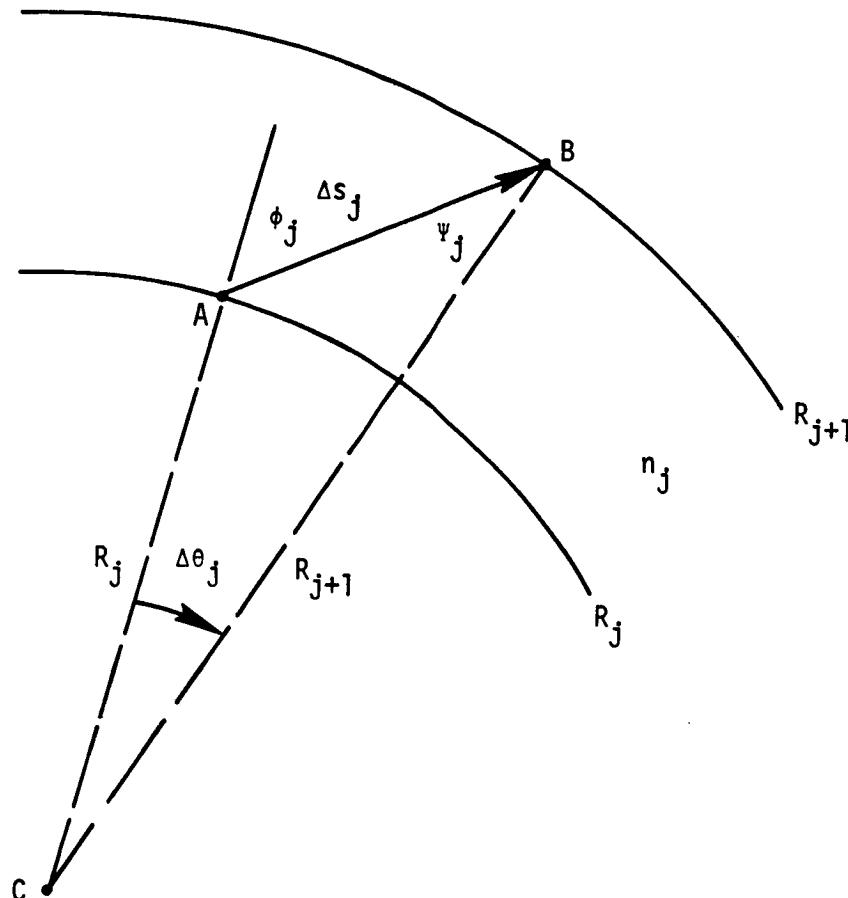


Figure 4-1.- The ray path in jth layer.

The equations presented in section 3.0 show that (i = initial, $j = 1$)

$$\phi_j = \arcsin \frac{n_i R_i \cos E_{Mi}}{n_j R_j} \quad (4.1)$$

and

$$\psi_j = \arcsin \frac{n_i R_i \cos E_{Mi}}{n_j R_{j+1}} \quad (4.2)$$

From the ABC triangle in figure 4-1

$$\Delta\theta_j + (180 - \phi_j) + \psi_j = 180$$

or

$$\Delta\theta_j = \phi_j - \psi_j \quad (4.3)$$

Substituting equations (4.1) and (4.2) into equation (4.3) yields

$$\Delta\theta_j = \arcsin \frac{n_i R_i \cos E_{Mi}}{n_j R_j} - \arcsin \frac{n_i R_i \cos E_{Mi}}{n_j R_{j+1}} \quad (4.4)$$

Note the trigonometric identity of

$$\arcsin X \pm \arcsin Y = \arcsin (X\sqrt{1-Y^2} \pm Y\sqrt{1-X^2}) \quad (4.5)$$

Using this identity $\Delta\theta_j$ becomes

$$\Delta\theta_j = \arcsin \frac{A}{R_i R_{i+1}} \frac{(R_{i+1}-R_i)(R_{i+1}+R_i)}{\sqrt{(R_{i+1}-A)(R_{i+1}+A)} + \sqrt{(R_i-A)(R_i+A)}} \quad (4.6)$$

where

$$A = \frac{1}{n_j} n_i R_i \cos E_{Mi} \quad (4.7)$$

Equation (4.6) should be left as is for computational accuracy. That is, do not make the substitution

$$(a - b)(a + b) = a^2 - b^2$$

For example, consider a 10-decimal digit computer, and let

$$a = 1234567899$$

$$b = 1234567898$$

Then

$$a^2 - b^2 = 2\ 000\ 000\ 000$$

But

$$(a - b)(a + b) = 2\ 469\ 135\ 797 \text{ (exactly)}$$

From figure 4-1 it is seen that the geometric path length in layer j is presented by the law of cosines as

$$\Delta s_j^2 = R_j^2 + R_{j+1}^2 - 2R_j R_{j+1} \cos \Delta\theta_j$$

or

$$\Delta s_j = \sqrt{(R_{j+1} - R_j)^2 + 2R_{j+1}R_j(1 - \cos \Delta\theta_j)} \quad (4.8)$$

However, equation (4.8) may be subject to severe round off problems since $\cos \Delta\theta_j$ will be close to 1. It is better to use the law of cosines to give

$$R_j^2 = \Delta s_j^2 + R_{j+1}^2 - 2\Delta s_j R_{j+1} \cos \psi_j$$

Solving for Δs_j gives

$$\Delta s_j = \frac{(R_{j+1} - R_j)(R_{j+1} + R_j)}{R_{j+1} \cos \psi_j + \sqrt{R_{j+1}^2 \cos^2 \psi_j - (R_{j+1} - R_j)(R_{j+1} + R_j)}} \quad (4.9)$$

where, from equation (4.2)

$$R_{j+1} \cos \psi_j = \frac{1}{n_j} \sqrt{n_j^2 R_{j+1}^2 - n_i^2 R_i^2 \cos^2 E_{Mi}} \quad (4.10)$$

The total optical path length (range) measured by a radar is

$$\rho_M = \sum_{j=1}^k n_j \Delta s_j \quad (n_1 = n_i = n_{\text{initial}}) \quad (4.11)$$

Also, the total central angle between the initial altitude and the final altitude is

$$\theta = \sum_{j=1}^k \Delta \theta_j \quad (4.12)$$

The equations for computing geometric range and geometric elevation angle may be derived by observing figure 4-2.

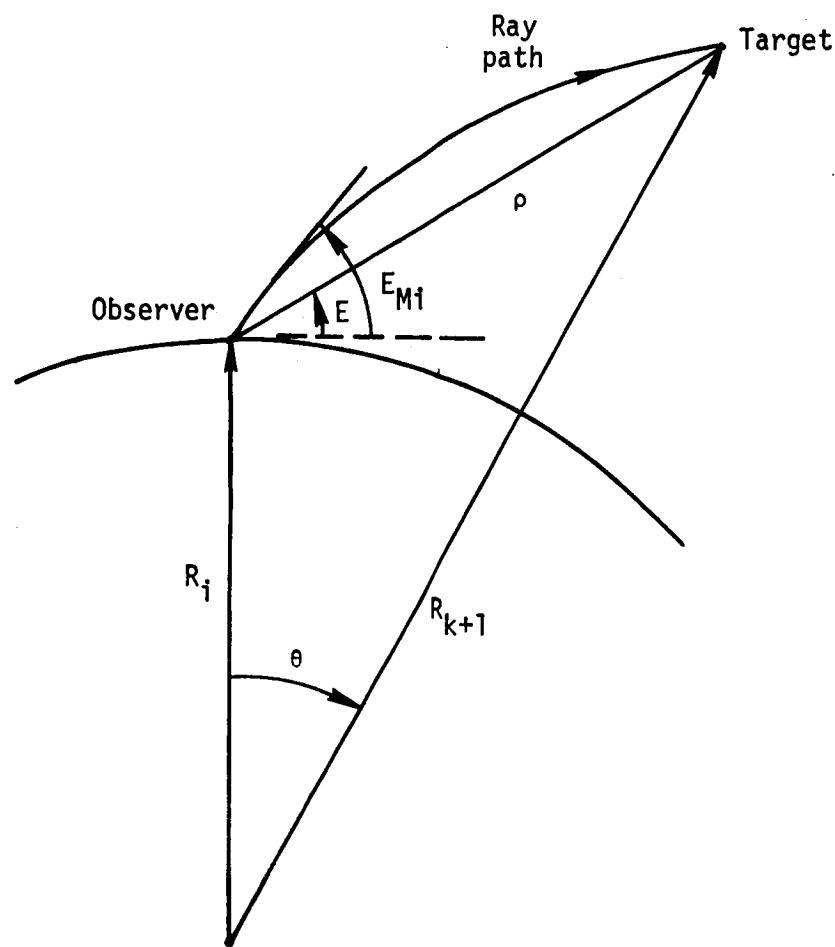


Figure 4-2.- Geometry for calculating ρ and E .

Let

$$T_1 = R_{k+1} \cos \theta - R_i \quad (4.13)$$

$$T_2 = R_{k+1} \sin \theta \quad (4.14)$$

Then

$$\boxed{\rho = \sqrt{T_1^2 + T_2^2}} \quad (4.15)$$

$$! \quad ! \\ ! \quad E = \arctan (T_1/T_2) \quad ! \quad (4.16) \\ ! \quad !$$

This formally concludes the derivation of the ray tracing equations. Note that no assumption has been made about how n varies with altitude. That is, an exponential atmosphere was not assumed. A method will now be suggested for picking values of R_j such that $R_{j+1}-R_j$ is not constant but $n_{j+1}-n_j = \Delta N$ will be constant. An exponential atmosphere is assumed.

$$N_j = N_o \exp(- (R_j - R_o)/H_s) \quad j=1,k \quad (4.17)$$

$$n_j = N_{j+1} \quad (4.18)$$

where $R_1 = R_i = R_{\text{initial}}$ is not necessarily equal to R_o . Given a desired value of ΔN (a small positive number) between layers,

$$N_j = N_i - (j-1) \Delta N = N_o \exp(- (R_j - R_o)/H_s)$$

Solving for R_j gives

$$! \quad ! \\ ! \quad R_j = R_o - H_s \ln \left[\frac{N_i}{N_o} - (j-1) \frac{\Delta N}{N_o} \right] \quad j = 1, k \quad ! \quad (4.19) \\ ! \quad ! \\ ! \quad R_j = R_{k+1} = R_i + H \quad \text{for } j = k+1 \quad ! \quad ! \\ ! \quad ! \\ ! \quad N_i = N_o \exp(- (R_i - R_o)/H_s) \quad R_1 = R_i \quad ! \quad (4.20) \\ ! \quad !$$

Calculate a value of ΔN by first calculating N_{k+1} , the value of N at R_{k+1} . Then calculate ΔN by dividing $N_i - N_{k+1}$ by the number of layers, k , that are desired. For high-altitude work, the following values of k are recommended:

$k = 50\ 000$ for $0 \leq E_{Mi} \leq 0.5^\circ$

$k = 5000$ for $0.5 \leq E_{Mi} \leq 3^\circ$

$k = 500$ for $E_{Mi} > 3^\circ$

The j th layer extends from R_j to R_{j+1} . Instead of using N_j (eq. (4.17)), it is better to use the average value of N , \bar{N}_j , for the j th layer.

$$\bar{N}_j = \frac{N_0}{R_{j+1} - R_j} \int_{R_j}^{R_{j+1}} \exp(-(r - R_0)/H_S) dr$$

or

$$\boxed{\bar{N}_j = \frac{H_S \Delta N}{R_{j+1} - R_j}} \quad (4.21)$$

$$\boxed{n_j = \bar{N}_j + 1} \quad (4.22)$$

To conclude this section, the equations presented here work only for increasing altitudes; that is, for $R_{j+1} - R_j > 0$. If decreasing altitudes are encountered, then the methods in sections 5.0 or 6.0 should be used.

5.0 THE INTEGRAL EQUATIONS

In section 4.0, equations (4.11) and (4.12) for ρ_M and θ can be expressed as integrals by letting $R_{j+1} \rightarrow R_j$. That is, use

$$dr = R_{j+1} - R_j$$

$$2r = R_{j+1} + R_j$$

$$r^2 = R_{j+1} R_j$$

$$d\theta = \Delta\theta_j$$

$$ds = \Delta s_j$$

Equation (4.6) yields

$$d\theta = \frac{A}{r^2} \frac{dr(2r)}{\sqrt{r^2 - A^2} + \sqrt{r^2 - A^2}} = \frac{A dr}{r \sqrt{r^2 - A^2}}$$

where

$$A = \frac{1}{n} n_i R_i \cos E_{Mi}$$

Thus

$$\theta = n_i R_i \cos E_{Mi} \int_{R_i}^{R_f} \frac{dr}{r \sqrt{n^2 r^2 - n_i^2 R_i^2 \cos^2 E_{Mi}}}$$

(5.1)

From equations (4.9) and (4.10),

$$ds = \frac{2r dr}{R \cos \psi + \sqrt{R^2 \cos^2 \psi}} = \frac{r dr}{R \cos \psi}$$

where

$$R \cos \psi = \frac{1}{n} \sqrt{n^2 R^2 - n_i^2 R_i^2 \cos^2 E_{Mi}}$$

From equation (4.11)

$$\rho_M = \int n ds$$

so that

$$\rho_M = \int_{R_i}^{R_f} \frac{n^2 r dr}{\sqrt{n^2 R^2 - n_i^2 R_i^2 \cos^2 E_{Mi}}} \quad (5.2)$$

and from equations (4.15) and (4.16)

$$T_1 = R_f \cos \theta - R_i \quad (5.3)$$

$$T_2 = R_f \sin \theta \quad (5.4)$$

$$\rho = \sqrt{T_1^2 + T_2^2} \quad (5.5)$$

$$E = \arctan (T_1/T_2) \quad (5.6)$$

Unfortunately, this does not conclude this simple derivation. The equations for θ and ρ_M are fraught with numerical evaluation difficulties. A set of more suitable equations will now be derived, which makes use of the spherical form of Snell's law.

Figure 5-1 shows a differential element of the ray path.

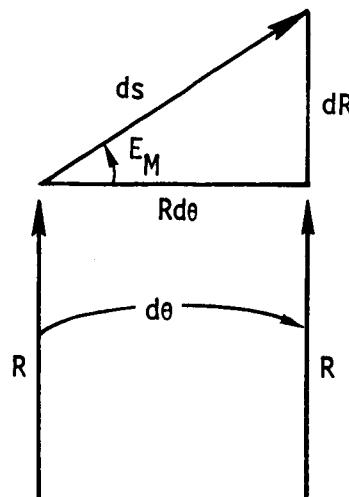


Figure 5-1.- Differential ray path.

Figure 5-1 shows that

$$d\theta = \frac{dR}{R \tan E_M} \quad (5.7)$$

The spherical form of Snell's law was given in equation (3.5) as

$$n R \cos E_M = n_i R_i \cos E_{Mi} \quad (5.8)$$

Taking differentials show that

$$dn R \cos E_M + n dR \cos E_M - n R dE_M \sin E_M = 0$$

Solving for dR yields

$$dR = R \tan E_M dE_M - \frac{R}{n} dn \quad (5.9)$$

Substituting this into equation (5.7) gives

$$d\theta = dE_M - \frac{dn}{n \tan E_M} \quad (5.10)$$

where, from equation (5.8)

$$\tan E_M = \frac{\operatorname{sgn}(E_{Mi})}{n_i R_i \cos E_{Mi}} \sqrt{n^2 R^2 - n_i^2 R_i^2 \cos^2 E_{Mi}} \quad (5.11)$$

Thus

$$\theta = E_{Mf} - E_{Mi} - \operatorname{sgn}(E_{Mi}) \cos E_{Mi} \int_{n_i}^{n_f} \frac{dn}{n \sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.12)$$

where

$$q = \frac{nR}{n_i R_i} \quad (5.13)$$

$$q_f = \frac{n_f R_f}{n_i R_i} \quad (5.14)$$

and where, from equation (5.8)

$$E_{Mf} = \operatorname{sgn}(E_{Mi}) \arccos \left(\frac{1}{q_f} \cos E_{Mi} \right) \quad (5.15)$$

Note that the sign of equations (5.11) and (5.15) is only correct if E_M does not undergo a sign change. This possibility exists for small negative values of E_{Mi} .

The equation for ρ_M will now be obtained as

$$d\rho_M = nds$$

where, from figure 5-1

$$ds = \frac{dR}{\sin E_M}$$

Thus

$$d\rho_M = \frac{ndR}{\sin E_M}$$

Substituting for dR , given in equation (5.9), gives

$$d\rho_M = \frac{nR}{\cos E_M} dE_M - \frac{R}{\sin E_M} dn$$

But from the spherical form of Snell's law,

$$nR = n_i R_i \cos E_{Mi} / \cos E_M$$

and from equation (5.8)

$$\sin E_M = \frac{\operatorname{sgn}(E_{Mi})}{nR} \sqrt{n^2 R^2 - n_i^2 R_i^2 \cos^2 E_{Mi}}$$

Thus

$$d\rho_M = n_i R_i \cos E_{Mi} \frac{dE_M}{\cos^2 E_M} - \frac{nR^2 \operatorname{sgn}(E_{Mi}) dn}{\sqrt{n^2 R^2 - n_i^2 R_i^2 \cos^2 E_{Mi}}} \quad (5.16)$$

Note that

$$\int_{E_{Mi}}^{E_{Mf}} \frac{dE_M}{\cos^2 E_M} = \tan E_{Mf} - \tan E_{Mi} \quad (5.17)$$

From equation (5.11)

$$\tan E_{Mf} = \frac{\text{sgn}(E_{Mi})}{n_i R_i \cos E_{Mi}} \sqrt{n_f^2 R_f^2 - n_i^2 R_i^2 \cos^2 E_{Mi}} \quad (5.18)$$

Combining equations (5.16), (5.17), and (5.18) gives

$$\rho_M = \frac{(n_f R_f - n_i R_i)(q_f + 1)}{\text{sgn}(E_{Mi}) \sqrt{q_f^2 - \cos^2 E_{Mi}} + \sin E_{Mi}}$$

$$= \text{sgn}(E_{Mi}) \int_{n_i}^{n_f} \frac{qR dn}{\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.19)$$

where, for accuracy

$$n_f R_f - n_i R_i = n_f h_f - n_i h_i - (N_i - N_f) R_o \quad (5.20)$$

where h is altitude above R_o and $N = n - 1$ is the modulus of refraction.

The equations will now be modified for Gaussian quadrature. Let

$$n = n_i - (n_i - n_f) x \quad (5.21)$$

or, for accuracy

$$N = N_i - (N_i - N_f) x \quad (n = N + 1) \quad (5.22)$$

Then $n = n_i$ for $x = 0$ and $n = n_f$ for $x = 1$ and

$$dn = - (N_i - N_f) dx$$

Changing the variable of integration yields

$$\begin{aligned} \theta &= \operatorname{sgn}(E_{Mi}) \arccos \left(\frac{1}{q_f} \cos E_{Mi} \right) - E_{Mi} \\ &+ (N_i - N_f) \operatorname{sgn}(E_{Mi}) \cos E_{Mi} \int_0^1 \frac{dx}{n \sqrt{q^2 - \cos^2 E_{Mi}}} \end{aligned} \quad (5.23)$$

$$\begin{aligned} \rho_M &= \frac{(n_f h_f - n_i h_i - (N_i - N_f) R_o) (q_f + 1)}{\operatorname{sgn}(E_{Mi}) \sqrt{q_f^2 - \cos^2 E_{Mi}} + \sin E_{Mi}} \\ &+ (N_i - N_f) \operatorname{sgn}(E_{Mi}) \int_0^1 \frac{q R dx}{\sqrt{q^2 - \cos^2 E_{Mi}}} \end{aligned} \quad (5.24)$$

where, as shown previously

$$q = \frac{nR}{n_i R_i} \quad q_f = \frac{n_f R_f}{n_i R_i} \quad (5.25)$$

$$N = N_i - (N_i - N_f)x \quad (n = N + 1) \quad (5.26)$$

Note that, in deriving equations (5.23) and (5.24), no assumption was made about $R = R_o + h$ as a function of n . That is, an exponential atmosphere was not assumed; for example, values of R and n could be obtained from an experimental look-up table. The integrands in the original equations for θ and ρ_M

(eqs. (5.1) and (5.2)) would need to be evaluated at several hundred or thousands of points for high-altitude work. The integrands in equations (5.23) and (5.24) need be evaluated at only a few points. Only five points, used later in this section, give high accuracy for $E_{Mi} \geq 0.7$ degree.

Equations (5.23) and (5.24) could now be evaluated using five-point Gaussian quadrature. However, there is a problem when E_{Mi} is close to zero. When $E_{Mi} = 0$, the integrands are infinite at $x = 0$. Note, however, that the integrals are finite. This causes an accuracy problem when using numerical quadrature. The use of more quadrature points (up to nine) alleviates the problem but is still not satisfactory. The difficulty disappears for $E_{Mi} \geq 0.7$ degrees. An alternate (to more quadrature points) solution to the problem can be obtained by splitting the integrals into two parts: the integral from $x = 0$ to $x = \epsilon$ (small), plus the integral from ϵ to one. An approximate analytic solution is obtained for the integrals from zero to ϵ . The integrals from ϵ to one are evaluated using Gaussian quadrature. The optimum value of ϵ has been determined to be approximately 0.06.

Define the following integrals as

$$I_{\theta A} = \text{sgn } (E_{Mi})(N_i - N_f) \cos E_{Mi} \int_0^{\epsilon} \frac{dx}{\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.27)$$

$$I_{\theta B} = \text{sgn } (E_{Mi})(N_i - N_f) \cos E_{Mi} \int_{\epsilon}^1 \frac{dx}{\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.28)$$

$$I_{\rho A} = \text{sgn } (E_{Mi})(N_i - N_f) \int_0^{\epsilon} \frac{qR dx}{\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.29)$$

$$I_{\rho B} = \text{sgn } (E_{Mi})(N_i - N_f) \int_{\epsilon}^1 \frac{qR dx}{\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.30)$$

$$I_\epsilon = \text{sgn } (E_{Mi})(N_i - N_f) \int_0^\epsilon \frac{dx}{\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.31)$$

Then for ϵ small

$$I_{\theta A} = \frac{1}{n} \cos E_{Mi} I_\epsilon \quad (5.32)$$

$$I_{\rho A} = \bar{q} \bar{R} I_\epsilon \quad (5.33)$$

where \bar{n} , \bar{q} and $\bar{R} = R_o + \bar{h}$ are evaluated at $\epsilon/2$.

An exponential atmosphere will now be assumed:

$$N = N_o \exp(-h/H_s) \quad (5.34)$$

where N_o is the modulus of refraction at the radius R_o and h is altitude above R_o . Solving for h gives

$$h = H_s \ln \frac{N_o}{N} \quad (5.35)$$

Note that

$$N_i = N_o \exp(-h_i/H_s) \quad (n_i = N_i + 1) \quad (5.36)$$

$$N_f = N_o \exp(-h_f/H_s) \quad (n_f = N_f + 1) \quad (5.37)$$

$$\bar{N} = N_i - (N_i - N_f) \epsilon/2 \quad (\bar{n} = \bar{N} + 1) \quad (5.38)$$

$$\bar{h} = H_s \ln \frac{N_o}{\bar{N}} \quad (5.39)$$

$$\bar{R} = R_O + \bar{h} \quad (5.40)$$

$$\bar{q} = \frac{n_i R}{n_i R_i} \quad (R_i = R_O + h_i) \quad (5.41)$$

An expansion of q in the integrand of I_E will now be obtained:

$$q = \frac{n(R_O + h)}{n_i R_i} = \frac{(n_i - (N_i - N_f)x)(R_O + h)}{n_i R_i}$$

or

$$q = \frac{1}{R_i} \left(1 - \frac{N_i - N_f}{n_i} x \right) (R_O + h) \quad (5.42)$$

An expansion for h in powers of x will now be obtained.

$$h = H_S \ln \frac{N_O}{N} = H_S \ln \frac{N_O}{N_i - (N_i - N_f)x}$$

or

$$h = H_S \ln \frac{\frac{N_O}{N_i}}{1 - \frac{N_i - N_f}{N_i} x}$$

Thus

$$h = H_S \ln \frac{N_O}{N_i} + H_S \ln \frac{1}{1 - \frac{N_i - N_f}{N_i} x}$$

But

$$\ln \frac{1}{1-y} = y + \frac{1}{2} y^2 + \frac{1}{3} y^3 + \frac{1}{4} y^4 + \dots$$

and

$$H_S \ln \frac{N_O}{N_i} = h_i$$

Thus

$$h = h_i + H_S \frac{N_i - N_f}{N_i} x + \frac{1}{2} H_S \left(\frac{N_i - N_f}{N_i} \right)^2 x^2 \quad (5.43)$$

Note that for the expansion of h to be accurate, h_i must satisfy

$$\boxed{\begin{array}{l} ! \\ ! h_i \leq H_S + h_f ! \\ ! \end{array}} \quad (5.44)$$

That is, ϵ should be set to zero if $h_i > H_S + h_f$.

Let

$$d = \frac{N_i - N_f}{N_i} \quad (5.45)$$

Then

$$\frac{N_i - N_f}{n_i} = \frac{N_i}{n_i} d \quad (5.46)$$

Equation (5.42) for q becomes

$$q = \left(1 - \frac{N_i}{n_i} dx \right) \left(1 + \frac{H_S}{R_i} d x + \frac{1}{2} \frac{H_S}{R_i} d^2 x^2 \right)$$

or

$$q = 1 + \left(\frac{H_S}{R_i} - \frac{N_i}{n_i} d \right) x + \frac{H_S}{R_i} \left(\frac{1}{2} - \frac{N_i}{n_i} \right) d^2 x^2 + \dots \quad (5.47)$$

Thus

$$q^2 - \cos^2 E_{Mi} = a x^2 + b x + \sin^2 E_{Mi} \quad (5.48)$$

where

$$a = d^2 \left[\left(\frac{H_S}{R_i} - \frac{N_i}{n_i} \right)^2 + \frac{H_S}{R_i} \left(1 - 2 \frac{N_i}{n_i} \right) \right] \quad (5.49)$$

$$b = 2d \left(\frac{H_S}{R_i} - \frac{N_i}{n_i} \right) \quad (5.50)$$

so

$$\int_0^\epsilon \frac{dx}{\sqrt{q^2 - \cos^2 E_{Mi}}} = \int_0^\epsilon \frac{dx}{\sqrt{ax^2 + bx + \sin^2 E_{Mi}}}$$

which may be evaluated analytically to give

$$\int_0^{\varepsilon} = \frac{1}{\sqrt{a}} \ln (2\sqrt{a(a\varepsilon^2 + b\varepsilon + \sin^2 E_{M1})} + 2a\varepsilon + b)$$

$$- \frac{1}{\sqrt{a}} \ln (2\sqrt{a \sin^2 E_{M1}} + b)$$

or

$$\int_0^{\varepsilon} = \frac{1}{\sqrt{a}} \ln \frac{2\sqrt{a(a\varepsilon^2 + b\varepsilon + \sin^2 E_{M1})} + 2a\varepsilon + b}{2\sqrt{a \sin^2 E_{M1}} + b} \quad (5.51)$$

Now let

$$b^* = \frac{H_S}{R_i} - \frac{N_i}{n_i} \quad (5.52)$$

$$a^* = (b^*)^2 + \frac{H_S}{R_i} \left(1 - 2 \frac{N_i}{n_i} \right) \quad (5.53)$$

Then

$$a = d^2 a^* \quad (5.54)$$

and

$$b = 2db^* \quad (5.55)$$

where

$$d = \frac{N_i - N_f}{N_i} \quad (5.56)$$

Now substitute equations (5.54), (5.55), and (5.56) into equation 5.51.

$$\int_0^\epsilon = \frac{N_i}{|N_i - N_f|} \frac{1}{\sqrt{a^*}} \ln$$

$$\frac{|d|\sqrt{a^*(a\epsilon^2 + b\epsilon + \sin^2 E_{Mi}) + d^2 a^*\epsilon + db^*}}{|d|\sqrt{a^* \sin^2 E_{Mi}} + db^*}$$

or

$$\int_0^\epsilon = \frac{N_i \operatorname{sgn}(E_{Mi})}{N_i - N_f} \frac{1}{\sqrt{a^*}} \ln$$

$$\frac{\sqrt{a^*(a\epsilon^2 + b\epsilon + \sin^2 E_{Mi})} + |d| a^*\epsilon + \operatorname{sgn}(E_{Mi}) b^*}{\sqrt{a^* \sin^2 E_{Mi}} + \operatorname{sgn}(E_{Mi}) b^*}$$

And

$$I_\epsilon = \operatorname{sgn}(E_{Mi}) (N_i - N_f) \int_0^\epsilon \frac{dx}{\sqrt{q^2 - \cos^2 E_{Mi}}}$$

becomes

$$I_\epsilon = \frac{N_i}{\sqrt{a^*}} \ln$$

$$\frac{\sqrt{a^*(a\epsilon^2 + b\epsilon + \sin^2 E_{Mi})} + |d| a^*\epsilon + \operatorname{sgn}(E_{Mi}) b^*}{\sqrt{a^* \sin^2 E_{Mi}} + \operatorname{sgn}(E_{Mi}) b^*}$$

(5.57)

All that remains is to set up the integrals $I_{\theta B}$ and $I_{\rho B}$ for Gaussian quadrature. The variable of integration x is changed to

$$x = \varepsilon + (1 - \varepsilon) y \quad (5.58)$$

$$dx = (1 - \varepsilon) dy \quad (5.59)$$

From equation (5.22)

$$\frac{N = N_i - (N_i - N_f) (\varepsilon + (1 - \varepsilon) y)}{!} \quad (n = N + 1) \quad (5.60)$$

And for an exponential atmosphere,

$$\frac{h = H_S \ln \frac{N_o}{N}}{!} \quad (5.61)$$

Then

$$I_{\theta B} = \text{sgn } (E_{Mi}) (N_i - N_f) (1 - \varepsilon) \cos E_{Mi} \int_0^1 \frac{dy}{n\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.62)$$

$$I_{\rho B} = \text{sgn } (E_{Mi}) (N_i - N_f) (1 - \varepsilon) \int_0^1 \frac{qR dy}{\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.63)$$

It has been determined that five-point Gaussian quadrature is adequate for most work. The five-point Gaussian quadrature points with their weights are shown as follows:

i	Y _i	W _i
1	0.04691 00770 30668	0.11846 34425 28095
2	0.23076 53449 47158	0.23931 43352 49683
3	0.5	0.28444 44444 44444
4	0.76923 46550 52842	0.23931 43352 49683
5	0.95308 99229 69332	0.11846 34425 28095

The equations in this section will now be summarized, assuming an exponential atmosphere. Inputs to the equations are:

N_O = modulus of refraction at radius R_O

H_S = atmospheric scale height

R_O = radius of Earth, 6 378 165 meters (value not critical)

h_i = initial altitude above R_O, observer altitude

h_f = final altitude above R_O, target altitude

E_{Mi} = elevation angle measured at h_i

ε ≈ 0.06, small initial integration interval

The equations are

$$N_i = N_O \exp(-h_i/H_S) \quad (n_i = N_i + 1) \quad (5.64)$$

$$N_f = N_O \exp(-h_f/H_S) \quad (n_f = N_f + 1) \quad (5.65)$$

$$R_i = R_O + h_i \quad (5.66)$$

$$R_f = R_O + h_f \quad (5.67)$$

$$d = (N_i - N_f)/N_i \quad (5.68)$$

$$b^* = \frac{H_S}{R_i} - \frac{N_i}{n_i} \quad (5.69)$$

$$a^* = (b^*)^2 + \frac{H_S}{R_i} \left(1 - 2 \frac{N_i}{n_i} \right) \quad (5.70)$$

$$a = d^2 a^* \quad (5.71)$$

$$b = 2d b^* \quad (5.72)$$

$$I_\epsilon = \frac{N_i}{\sqrt{a^*}} \ell n$$

$$\frac{\sqrt{a^*(ae^2 + be + \sin^2 E_{Mi})} + |d| a^* \epsilon + \text{sgn}(E_{Mi}) b^*}{\sqrt{a^* \sin^2 E_{Mi}} + \text{sgn}(E_{Mi}) b^*} \quad (5.73)$$

$$\bar{N} = N_i - (N_i - N_f) \epsilon / 2 \quad (5.74)$$

$$\bar{n} = \bar{N} + 1 \quad (5.75)$$

$$\bar{h} = H_S \ell n (N_o / \bar{N}) \quad (5.76)$$

$$\bar{R} = R_o + \bar{h} \quad (5.77)$$

$$\bar{q} = \frac{\bar{n} - \bar{R}}{n_i R_i} \quad (5.78)$$

$$I_{\theta A} = \frac{1}{n} \cos E_{Mi} I_\epsilon \quad \text{radians} \quad (5.79)$$

$$I_{\rho A} = \bar{q} \bar{R} I_\epsilon \quad (5.80)$$

$$I_{\theta B} = \text{sgn}(E_{Mi})(N_i - N_f)(1 - \epsilon) \cos E_{Mi} \int_0^1 \frac{dy}{\sqrt{n^2 q^2 - \cos^2 E_{Mi}}} \quad \text{radians} \quad (5.81)$$

$$I_{\rho B} = \text{sgn}(E_{Mi}) (N_i - N_f)(1 - \epsilon) \int_0^1 \frac{qR dy}{\sqrt{q^2 - \cos^2 E_{Mi}}} \quad (5.82)$$

where

$$N = N_i - (N_i - N_f) (\epsilon + (1 - \epsilon) y) \quad (5.83)$$

$$n = N + 1 \quad (5.84)$$

$$h = H_S \ln (N_o/N) \quad (5.85)$$

$$R = R_o + h \quad (5.86)$$

$$q = \frac{nR}{n_i R_i} \quad (5.87)$$

The values θ and ρ_M are now given by

$$q_f = \frac{n_f R_f}{n_i R_i} \quad (5.88)$$

$$\theta = \text{sgn}(E_{Mi}) \arccos \left(\frac{1}{q_f} \cos E_{Mi} \right) - E_{Mi} + I_{\theta A} + I_{\theta B} \quad (5.89)$$

$$\rho_M = \frac{(n_f h_f - n_i h_i - (N_i - N_f) R_o)(q_f + 1)}{\text{sgn}(E_{Mi}) \sqrt{q_f^2 - \cos^2 E_{Mi}} + \sin E_{Mi}} + I_{\rho A} + I_{\rho B} \quad (5.90)$$

$$T_1 = R_f \cos \theta - R_i \quad (5.91)$$

$$T_2 = R_f \sin \theta \quad (5.92)$$

$$\rho = \sqrt{T_1^2 + T_2^2} \quad (5.93)$$

$$E = \arctan (T_1/T_2) \quad (5.94)$$

$$\Delta\rho_7 = \rho_M - \rho \quad (5.95)$$

$$\Delta E_{18} = E_{Mi} - E \quad (5.96)$$

The subscript 7 is used because this was the seventh algorithm investigated that gave the range refraction correction. The subscript 18 is used because this was the eighteenth algorithm investigated that gave the elevation angle refraction correction.

The accuracies of $\Delta\rho_7$ and ΔE_{18} are illustrated in tables 5-1 through 5-18. The precise values of $\rho_M - \rho$ and $E_{Mi} - E$ were obtained from the appendixes. For these tables, $h_i = 0$ unless otherwise noted.

TABLE 5-1.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_o = 0.000395$, $H_S = 5446$ METERS, $h_f = 10^2$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	19.12	18.42	19.12	1.758	1.541	1.755
.1	12.62	12.62	12.62	1.161	1.161	1.161
.2	8.82	8.81	8.82	.811	.811	.811
.3	6.58	6.58	6.58	.606	.606	.606
.4	5.19	5.19	5.19	.478	.478	.478
.5	4.26	4.26	4.26	.392	.392	.392
.7	3.12	3.12	3.12	.287	.287	.287
1	2.21	2.21	2.21	.204	.204	.204
2	1.12	1.12	1.12	.103	.103	.103
3	.75	.75	.75	.069	.069	.069
4	.56	.56	.56	.051	.051	.051
5	.45	.45	.45	.041	.041	.041
7	.32	.32	.32	.029	.029	.029
10	.23	.23	.23	.020	.020	.020

TABLE 5-2.- $\Delta\varphi_7$ AND ΔE_{18} FOR $N_O = 0.000395$,
 $H_S = 5446$ METERS, $h_f = 10^4$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\varphi_7$ $\epsilon = 0$	$\Delta\varphi_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	121.25	115.62	121.47	12.922	11.358	12.902
.1	112.03	109.85	112.11	12.069	11.405	12.051
.2	104.01	103.29	104.04	11.310	11.070	11.299
.3	96.97	96.76	96.98	10.631	10.553	10.625
.4	90.74	90.68	90.75	10.019	9.995	10.016
.5	85.19	85.17	85.20	9.465	9.459	9.464
.7	75.74	75.74	75.74	8.503	8.503	8.503
1	64.63	64.63	64.64	7.343	7.343	7.343
2	42.46	42.46	42.46	4.924	4.924	4.924
3	31.06	31.06	31.06	3.631	3.631	3.631
4	24.29	24.29	24.29	2.848	2.848	2.848
5	19.86	19.86	19.86	2.331	2.331	2.331
7	14.50	14.50	14.50	1.699	1.699	1.699
10	10.29	10.29	10.29	1.199	1.199	1.199

TABLE 5-3.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_o = 0.000395$, $H_S = 5446$ METERS, $h_f = 10^6$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	142.1	133.9	141.9	20.65	18.66	20.63
.1	130.4	126.8	130.1	19.34	18.45	19.33
.2	120.5	118.9	120.1	18.18	17.84	18.17
.3	111.9	111.0	111.5	17.14	17.02	17.14
.4	104.3	103.8	103.9	16.20	16.17	16.21
.5	97.7	97.2	97.3	15.35	15.35	15.36
.7	86.6	86.1	86.1	13.88	13.89	13.89
1	73.7	73.2	73.3	12.10	12.10	12.10
2	48.6	48.2	48.2	8.34	8.35	8.35
3	35.9	35.5	35.5	6.28	6.29	6.29
4	28.3	27.9	27.9	5.00	5.00	5.00
5	23.2	22.9	22.9	4.13	4.14	4.14
7	17.1	16.8	16.8	3.05	3.05	3.05
10	12.2	11.9	12.0	2.17	2.17	2.17

TABLE 5-4.- $\Delta\phi_7$ AND ΔE_{18} FOR $N_O = 0.000395$,
 $H_S = 5446$ METERS, $h_f = 10^8$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\phi_7$ $\epsilon = 0$	$\Delta\phi_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	144.1	135.7	143.9	21.66	19.60	21.64
.1	132.1	128.4	131.8	20.25	19.34	20.24
.2	121.8	120.2	121.4	19.00	18.65	19.00
.3	113.0	112.1	112.6	17.89	17.77	17.89
.4	105.3	104.7	104.8	16.89	16.85	16.89
.5	98.5	98.0	98.0	15.98	15.98	15.99
.7	87.1	86.6	86.7	14.41	14.42	14.42
1	74.1	73.6	73.6	12.52	12.53	12.53
2	48.8	48.3	48.3	8.57	8.58	8.58
3	35.9	35.5	35.5	6.42	6.43	6.43
4	28.3	27.9	27.9	5.09	5.10	5.10
5	23.3	22.9	22.9	4.20	4.21	4.21
7	17.1	16.8	16.8	3.09	3.09	3.09
10	12.2	11.9	12.0	2.19	2.19	2.19

TABLE 5-5.- $\Delta\varphi_7$ AND ΔE_{18} FOR $N_o = 0.000395$,
 $H_S = 5446$ METERS, $\rho_M = 5 \cdot 10^4$ METERS, $h_i = 2000$ METERS

E_{Mi} , deg	h_f , meters	$\rho_m - \rho$, meters	$\Delta\varphi_7$ $\epsilon = 0$	$\Delta\varphi_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
-2	381.54	16.00	16.00	16.00	1.394	1.394	1.394
-1.5	819.49	15.35	15.35	15.35	1.356	1.356	1.356
-1	1257.46	14.72	14.72	14.72	1.319	1.319	1.319
-.5	1695.41	14.14	14.14	14.14	1.284	1.284	1.284
a-.15	2001.96	13.74	-14.75	-14.84	1.260	-.019	-.035
0	2133.32	13.58	13.23	13.58	1.250	1.082	1.248
.5	2571.15	13.05	13.05	13.05	1.217	1.217	1.217
1	3008.87	12.55	12.55	12.55	1.186	1.186	1.186
1.5	3446.45	12.07	12.07	12.07	1.156	1.156	1.156
2	3883.85	11.62	11.62	11.62	1.126	1.126	1.126
3	4758.02	10.79	10.79	10.79	1.071	1.071	1.071
4	5631.11	10.03	10.03	10.03	1.019	1.019	1.019
5	6502.87	9.35	9.35	9.35	.971	.971	.971
7	8241.41	8.18	8.18	8.18	.884	.884	.884
10	10832.90	6.80	6.80	6.80	.773	.773	.773

^a E_M undergoes a sign reversal.

TABLE 5-6.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_O = 0.000395$,
 $H_S = 5446$ METERS, $h_i = 10^6$ METERS, $h_f = 0$

E_{Mi} , deg	E_{Mf} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$
-30.2	-2.009	48.5	48.0	0.237	0.236
-30.3	-3.273	33.4	33.0	.131	.130
-30.4	-4.173	27.3	26.9	.093	.092
-30.5	-4.913	23.6	23.2	.073	.072
-30.6	-5.557	21.1	20.8	.061	.060
-30.8	-6.666	17.9	17.6	.046	.045
-31	-7.621	15.8	15.5	.037	.036
-31.5	-9.622	12.6	12.4	.025	.025
-32	-11.298	10.8	10.6	.020	.019
-32.5	-12.777	9.6	9.4	.016	.016
-33	-14.122	8.7	8.6	.014	.013
-34	-16.536	7.5	7.4	.011	.011
-35	-18.700	6.7	6.5	.009	.009
-37	-22.559	5.6	5.5	.007	.007
-40	-27.650	4.6	4.5	.005	.005
-45	-35.150	3.7	3.7	.003	.003
-50	-41.989	3.2	3.1	.003	.003
-55	-48.452	2.9	2.8	.002	.002
-60	-54.678	2.6	2.6	.002	.002
-70	-66.704	2.3	2.3	.001	.001
-90	-90	2.2	2.1	0	0

TABLE 5-7.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_O = 0.000325$, $H_S = 6735$ METERS, $h_f = 10^2$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	13.88	13.54	13.88	1.032	0.892	1.030
.1	9.61	9.61	9.61	.715	.714	.715
.2	6.93	6.93	6.93	.516	.516	.516
.3	5.27	5.27	5.27	.392	.392	.392
.4	4.20	4.20	4.20	.312	.312	.312
.5	3.47	3.47	3.47	.258	.258	.258
.7	2.55	2.55	2.55	.190	.190	.190
1	1.81	1.81	1.81	.135	.135	.135
2	.92	.92	.92	.068	.068	.068
3	.62	.62	.62	.046	.046	.046
4	.46	.46	.46	.034	.034	.034
5	.37	.37	.37	.027	.027	.027
7	.26	.26	.26	.020	.020	.020
10	.19	.19	.19	.014	.014	.014

TABLE 5-8.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_o = 0.000325$,
 $H_S = 6735$ METERS, $h_f = 10^4$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	93.66	91.01	93.89	8.138	7.094	8.125
.1	88.18	86.96	88.29	7.700	7.176	7.687
.2	83.21	82.71	83.27	7.297	7.063	7.288
.3	78.69	78.50	78.70	6.928	6.831	6.922
.4	74.57	74.50	74.59	6.587	6.550	6.584
.5	70.79	70.76	70.80	6.273	6.259	6.271
.7	64.11	64.11	64.12	5.712	5.711	5.712
1	55.89	55.89	55.89	5.011	5.011	5.011
2	38.18	38.18	38.18	3.463	3.463	3.463
3	28.41	28.41	28.41	2.589	2.589	2.589
4	22.40	22.40	22.40	2.045	2.045	2.045
5	18.41	18.41	18.41	1.680	1.680	1.680
7	13.50	13.50	13.50	1.230	1.230	1.230
10	9.61	9.61	9.61	.869	.869	.869

TABLE 5-9.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_o = 0.000325$,
 $H_s = 6735$ METERS, $h_f = 10^6$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	108.8	104.9	108.8	13.82	12.51	13.81
.1	102.4	100.3	102.2	13.14	12.44	13.13
.2	96.7	95.5	96.4	12.51	12.17	12.50
.3	91.5	90.7	91.1	11.93	11.78	11.93
.4	86.8	86.2	86.4	11.39	11.33	11.39
.5	82.4	82.0	82.1	10.90	10.88	10.90
.7	74.9	74.5	74.5	10.02	10.02	10.02
1	65.7	65.2	65.3	8.91	8.91	8.91
2	45.9	45.5	45.5	6.41	6.41	6.41
3	34.8	34.4	34.4	4.93	4.94	4.94
4	27.8	27.5	27.5	3.98	3.98	3.98
5	23.1	22.7	22.8	3.32	3.32	3.32
7	17.1	16.8	16.9	2.47	2.47	2.47
10	12.3	12.1	12.1	1.77	1.77	1.77

TABLE 5-10.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_0 = 0.000325$,
 $H_S = 6735$ METERS, $h_f = 10^8$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	109.8	105.8	109.8	14.52	13.18	14.51
.1	103.2	101.1	103.0	13.78	13.07	13.77
.2	97.4	96.2	97.1	13.10	12.76	13.10
.3	92.1	91.3	91.7	12.48	12.33	12.48
.4	87.3	86.7	86.9	11.91	11.85	11.91
.5	82.9	82.4	82.5	11.38	11.36	11.38
.7	75.2	74.8	74.8	10.44	10.44	10.44
1	65.9	65.5	65.5	9.25	9.26	9.26
2	46.0	45.5	45.6	6.61	6.61	6.61
3	34.8	34.4	34.4	5.06	5.07	5.07
4	27.8	27.5	27.5	4.07	4.07	4.07
5	23.1	22.8	22.8	3.38	3.39	3.38
7	17.2	16.8	16.9	2.51	2.51	2.51
10	12.3	12.1	12.1	1.79	1.79	1.79

TABLE 5-11.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_o = 0.000325$,
 $H_S = 6735$ METERS, $\rho_M = 5 \cdot 10^4$ METERS, $h_i = 2000$ METERS

E_{Mi} , deg	h_f , meters	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
-2	402.45	13.68	13.68	13.68	0.974	0.974	0.974
-1.5	839.59	13.23	13.23	13.23	.953	.953	.953
-1	1276.79	12.80	12.80	12.80	.932	.932	.932
-.5	1714.00	12.38	12.38	12.38	.912	.912	.912
a-.17	2002.56	12.12	-1697	-1703	.899	-.015	-.027
0	2151.20	11.99	11.77	11.99	.893	.767	.891
.5	2588.35	11.61	11.61	11.61	.874	.874	.874
1	3025.42	11.25	11.25	11.25	.855	.855	.855
1.5	3462.38	10.90	10.90	10.90	.837	.837	.837
2	3899.19	10.57	10.57	10.57	.820	.820	.820
3	4772.45	9.94	9.94	9.94	.787	.787	.787
4	5644.32	9.36	9.36	9.36	.755	.755	.755
5	6515.15	8.84	8.84	8.84	.725	.725	.725
7	8252.04	7.90	7.90	7.90	.670	.670	.670
10	10841.54	6.75	6.75	6.75	.598	.598	.598

^a E_M undergoes a sign reversal.

TABLE 5-12.- $\Delta\phi_7$ AND ΔE_{18} FOR $N_o = 0.000325$,
 $H_s = 6735$ METERS, $h_i = 10^6$ METERS, $h_f = 0$

E_{Mi} , deg	E_{Mf} , deg	$\rho_M - \rho$, meters	$\Delta\phi_7$ $\epsilon = 0$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$
-30.2	-1.891	47.5	47.1	0.221	0.220
-30.3	-3.202	33.1	32.7	.124	.124
-30.4	-4.118	27.2	26.8	.090	.089
-30.5	-4.866	23.6	23.3	.071	.071
-30.6	-5.516	21.2	20.9	.059	.059
-30.8	-6.632	18.0	17.7	.045	.045
-31	-7.591	15.9	15.6	.037	.036
-31.5	-9.598	12.8	12.6	.025	.025
-32	-11.278	11.0	10.8	.020	.019
-32.5	-12.760	9.8	9.6	.016	.016
-33	-14.106	8.9	8.7	.014	.014
-34	-16.523	7.6	7.5	.011	.011
-35	-18.688	6.8	6.6	.009	.009
-37	-22.549	5.7	5.6	.007	.007
-40	-27.642	4.7	4.6	.005	.005
-45	-35.144	3.8	3.7	.003	.003
-50	-41.985	3.3	3.2	.003	.003
-55	-48.449	2.9	2.9	.002	.002
-60	-54.676	2.7	2.6	.002	.002
-70	-66.702	2.4	2.3	.001	.001
-90	-90	2.2	2.1	0	0

TABLE 5-13.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_O = 0.000255$,
 $H_S = 7892$ METERS, $h_f = 10^2$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	10.18	10.01	10.18	0.646	0.553	0.645
.1	7.21	7.22	7.22	.458	.458	.458
.2	5.29	5.29	5.29	.336	.336	.336
.3	4.07	4.07	4.07	.258	.258	.258
.4	3.26	3.26	3.26	.207	.207	.207
.5	2.70	2.70	2.70	.171	.171	.171
.7	1.99	1.99	1.99	.127	.127	.127
1	1.42	1.42	1.42	.090	.090	.090
2	.72	.72	.72	.046	.046	.046
3	.48	.48	.48	.031	.031	.031
4	.36	.36	.36	.023	.023	.023
5	.29	.29	.29	.018	.018	.018
7	.21	.21	.21	.013	.013	.013
10	.15	.15	.15	.009	.009	.009

TABLE 5-14.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_o = 0.000255$, $H_S = 7892$ METERS, $h_f = 10^4$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	71.82	70.53	72.02	5.299	4.589	5.290
.1	68.19	67.54	68.30	5.045	4.659	5.036
.2	64.85	64.55	64.91	4.809	4.617	4.802
.3	61.75	61.62	61.78	4.589	4.501	4.584
.4	58.88	58.83	58.90	4.385	4.346	4.382
.5	56.21	56.19	56.23	4.194	4.177	4.192
.7	51.42	51.42	51.43	3.848	3.845	3.848
1	45.36	45.36	45.36	3.407	3.407	3.407
2	31.72	31.72	31.72	2.400	2.400	2.400
3	23.87	23.87	23.87	1.811	1.811	1.811
4	18.93	18.93	18.93	1.437	1.437	1.437
5	15.61	15.61	15.61	1.184	1.184	1.184
7	11.48	11.48	11.48	.869	.869	.869
10	8.19	8.19	8.19	.615	.615	.615

TABLE 5-15.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_O = 0.000255$,
 $H_S = 7892$ METERS, $h_f = 10^6$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	84.0	82.0	84.0	9.47	8.57	9.46
.1	80.0	78.7	79.9	9.06	8.54	9.06
.2	76.3	75.5	76.1	8.69	8.40	8.68
.3	72.9	72.3	72.6	8.34	8.19	8.34
.4	69.7	69.3	69.4	8.01	7.94	8.01
.5	66.8	66.4	66.5	7.71	7.68	7.71
.7	61.5	61.2	61.2	7.15	7.15	7.15
1	54.9	54.5	54.5	6.44	6.44	6.44
2	39.75	39.4	39.4	4.76	4.76	4.76
3	30.7	30.4	30.4	3.72	3.73	3.73
4	24.9	24.6	24.6	3.03	3.04	3.04
5	20.8	20.5	20.5	2.55	2.55	2.55
7	15.6	15.3	15.3	1.91	1.91	1.91
10	11.2	11.0	11.0	1.38	1.38	1.38

TABLE 5-16.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_o = 0.000255$,
 $H_S = 7892$ METERS, $h_f = 10^8$ METERS

E_{Mi} , deg	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
0	84.5	82.5	84.5	9.96	9.06	9.96
.1	80.4	79.1	80.3	9.53	9.00	9.52
.2	76.7	75.8	76.5	9.12	8.84	9.12
.3	73.2	72.6	72.9	8.75	8.60	8.74
.4	70.0	69.5	69.7	8.40	8.33	8.40
.5	67.1	66.6	66.7	8.07	8.04	8.07
.7	61.7	61.4	61.4	7.47	7.47	7.48
1	55.0	54.6	54.7	6.71	6.71	6.71
2	39.8	39.4	39.5	4.93	4.93	4.93
3	30.8	30.4	30.5	3.83	3.84	3.84
4	24.9	24.6	24.6	3.11	3.11	3.11
5	20.8	20.5	20.5	2.60	2.61	2.61
7	15.6	15.3	15.3	1.95	1.95	1.95
10	11.2	11.0	11.0	1.40	1.40	1.40

TABLE 5-17.- $\Delta\rho_7$ AND ΔE_{18} FOR $N_O = 0.000255$,
 $H_S = 7892$ METERS, $\rho_M = 5 \cdot 10^4$ METERS, $h_i = 2000$ METERS

E_{Mi} , deg	hf, meters	$\rho_M - \rho$, meters	$\Delta\rho_7$ $\epsilon = 0$	$\Delta\rho_7$ $\epsilon = 0.06$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$	ΔE_{18} $\epsilon = 0.06$
-2	417.44	11.00	11.00	11.00	0.673	0.673	0.673
-1.5	854.17	10.69	10.69	10.69	.660	.660	.660
-1	1290.97	10.39	10.39	10.39	.648	.648	.648
-.5	1727.80	10.10	10.10	10.10	.636	.636	.636
^a -1.19	1998.63	9.93	.08	.08	.629	.005	.005
0	2164.62	9.83	9.70	9.83	.625	.533	.623
.5	2601.41	9.56	9.56	9.56	.613	.613	.613
1	3038.13	9.31	9.31	9.31	.602	.602	.602
1.5	3474.74	9.06	9.06	9.06	.591	.591	.591
2	3911.22	8.82	8.82	8.82	.581	.581	.581
3	4783.64	8.37	8.37	8.37	.560	.560	.560
4	5655.11	7.95	7.95	7.95	.541	.541	.541
5	6525.38	7.56	7.56	7.56	.522	.522	.522
7	8261.25	6.86	6.86	6.86	.487	.487	.487
10	10849.43	5.97	5.97	5.97	.440	.440	.440

^a E_M undergoes a sign reversal.

TABLE 5-18.- $\Delta\phi_7$ AND ΔE_{18} FOR $N_o = 0.000255$,
 $H_S = 7892$ METERS, $h_i = 10^6$ METERS, $h_f = 0$

E_{Mi} , deg	E_{Mf} , deg	$\rho_M - \rho$, meters	$\Delta\phi_7$ $\epsilon = 0$	$E_{Mi} - E$, mrad	ΔE_{18} $\epsilon = 0$
-30.2	-1.766	42.6	42.2	0.191	0.191
-30.3	-3.130	29.8	29.5	.109	.109
-30.4	-4.062	24.6	24.3	.079	.079
-30.5	-4.819	21.4	21.1	.063	.063
-30.6	-5.474	19.3	19.0	.053	.053
-30.8	-6.597	16.4	16.1	.040	.040
-31	-7.561	14.5	14.3	.033	.033
-31.5	-9.575	11.7	11.5	.023	.023
-32	-11.258	10.1	9.9	.018	.018
-32.5	-12.742	8.9	8.8	.015	.015
-33	-14.090	8.1	8.0	.013	.012
-34	-16.509	7.0	6.9	.010	.010
-35	-18.677	6.2	6.1	.008	.008
-37	-22.540	5.2	5.1	.006	.006
-40	-27.635	4.3	4.2	.005	.004
-45	-35.139	3.5	3.4	.003	.003
-50	-41.980	3.0	2.9	.002	.002
-55	-48.445	2.7	2.6	.002	.002
-60	-54.673	2.5	2.4	.001	.001
-70	-66.700	2.2	2.1	.001	.001
-90	-90	2.0	2.0	0	0

6.0 THE RAY PATH DIFFERENTIAL EQUATIONS

Figure 6-1 shows the ray path geometry, from the initial altitude h_i to the final altitude h_f .

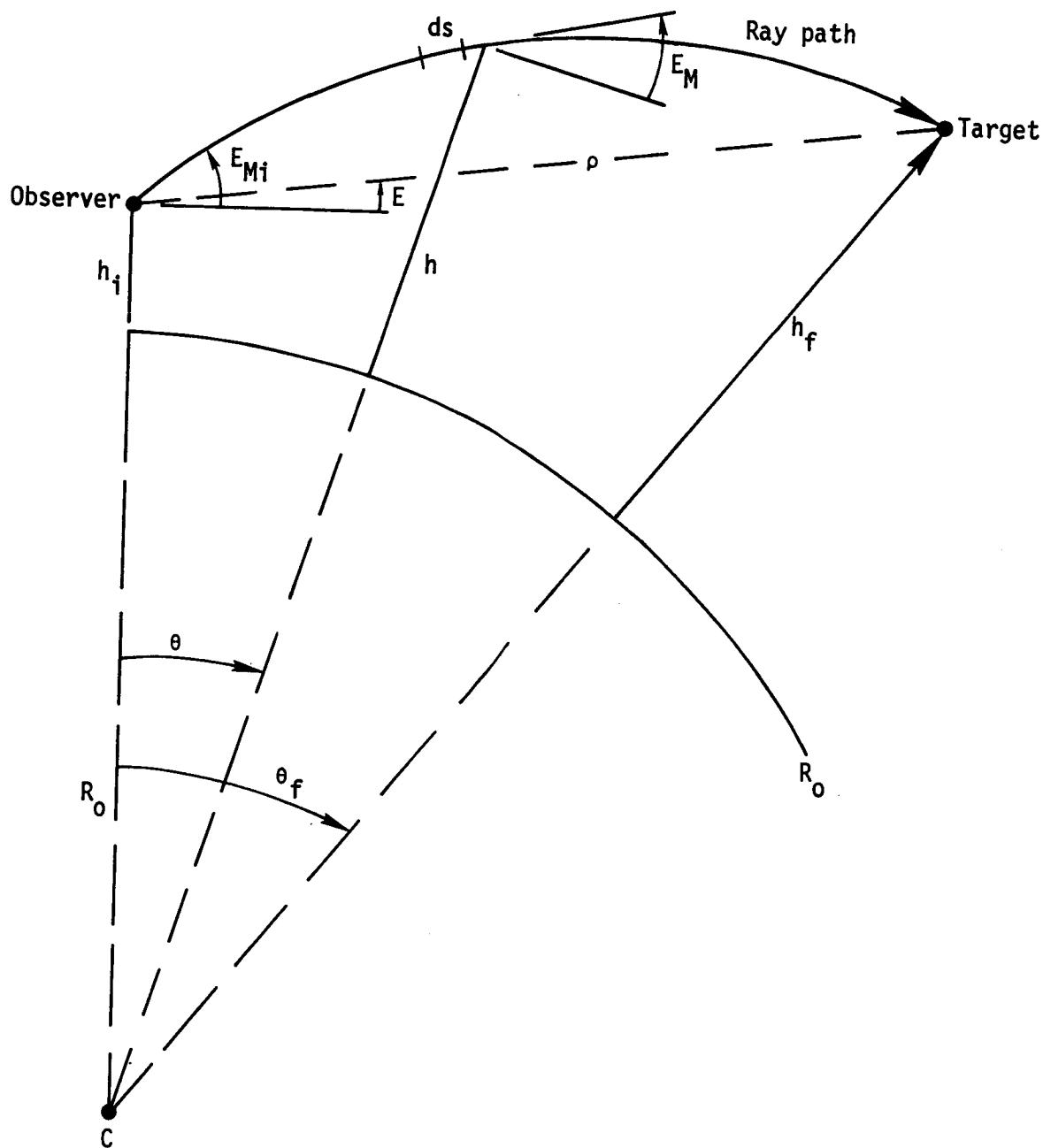


Figure 6-1.- Ray path geometry.

The following parameters are defined:

R_0 = reference radius of Earth = 6 378 165 meters (value not critical)
 $R = R_0 + h$

h = altitude above R_0
 h_i = initial altitude at observer
 h_f = final altitude at target

θ = central angle
 θ_i = zero
 θ_f = value at target

E_M = elevation angle of ray path
 E_{Mf} = value of E_M at target, final value
 E_{Mi} = initial value of E_M , measured by equipment

ds = differential path length

ρ = true, straight-line, geometric range

E = true, geometric elevation angle

c = speed of light in a vacuum

n = index of refraction at altitude h , equal to $c/(speed of light at h)$
 n_0 = value of n at R_0

ρ_M = measured range

$$\rho_M = \int n ds \quad (6.1)$$

The value $N = n - 1$ is the modulus of refraction. An exponential atmosphere will be assumed.

$$N = N_0 \exp(-h/H_S) \quad (6.2)$$

H_S = atmospheric scale height

Figure 6-2(a) shows the differential path length element ds and its two components. Figure 6-2(b) shows the photon velocity along ds and its two components.

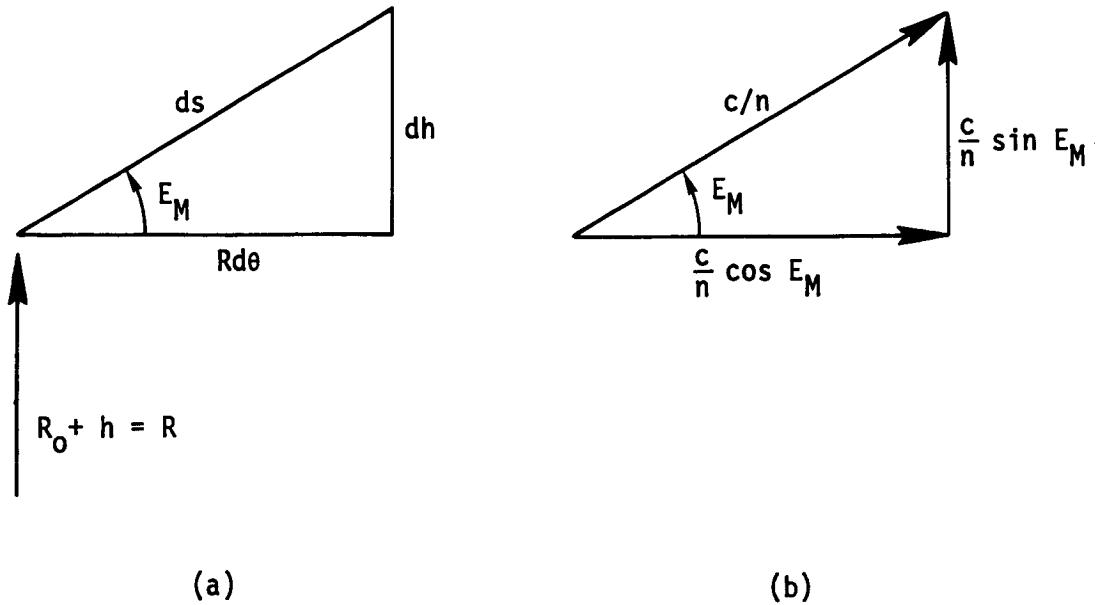


Figure 6-2.- Differential geometry.

As shown in figures 6-2(a) and (b),

$$\frac{ds}{dt} = \frac{c}{n} \quad (6.3)$$

$$\frac{dh}{dt} = \frac{c}{n} \sin E_M \quad (6.4)$$

$$\frac{d\theta}{dt} = \frac{1}{R} \frac{c}{n} \cos E_M \quad (6.5)$$

where, from equation (6.2)

$$n = 1 + N = 1 + N_0 \exp(-h/H_S) \quad (6.6)$$

Note from equation (6.1) that

$$dp_M = nds \quad (6.7)$$

And using equation (6.3)

$$\frac{dp_M}{dt} = c \quad (6.8)$$

An expression is now needed for E_M . Equation (3.5), the spherical form of Snell's law could be used.

$$nR \cos E_M = n_i R_i \cos E_{Mi} \quad (6.9)$$

This, along with equation (6.6), gives E_M as a function of h , and equations (6.4) and (6.5) can now be integrated. However, this approach has numerical difficulties. If $E_{Mi} = 0$, then h (from eq. (6.4)) will remain at its initial value of h_i . It has been determined that it is more accurate to develop a differential equation for E_M and integrate it.

Differentiating equation (6.9) with respect to time yields

$$\dot{n}R \cos E_M + nh \cos E_M - nR \dot{E}_M \sin E_M = 0$$

But, from equation (6.6)

$$\dot{n} = -\frac{N_0}{H_S} \dot{h} \exp(-h/H_S) = -\frac{N}{H_S} \dot{h}$$

Thus

$$-\frac{N}{H_S} \dot{h} \cos E_M + nh \cos E_M - nR \dot{E}_M \sin E_M = 0$$

Using equation (6.4) for \dot{h}

$$-R \frac{N}{H_S} \frac{c}{n} \sin E_M \cos E_M + c \sin E_M \cos E_M - nR \dot{E}_M \sin E_M = 0$$

And the equation for \dot{E}_M is

$$\frac{dE_M}{dt} = \left(\frac{1}{R} - \frac{N}{nH_S} \right) \frac{c}{n} \cos E_M \quad (6.10)$$

Now let

$$a = ct \quad (6.11)$$

Then $da = cdt$ and the resulting equations are summarized below.

$$\rho_M = a \quad (6.12)$$

$$\frac{dh}{da} = \frac{1}{n} \sin E_M \quad (6.13)$$

$$\frac{d\theta}{da} = \frac{1}{R} \frac{1}{n} \cos E_M \quad (6.14)$$

$$\frac{dE_M}{da} = \left(\frac{1}{R} - \frac{N}{nH_S} \right) \frac{1}{n} \cos E_M \quad (6.15)$$

The initial conditions are

$$a = 0$$

$$h = h_i$$

$$\theta = \theta_i = 0$$

$$E_M = E_{Mi}$$

And where

$$R = R_o + h \quad (6.16)$$

$$N = N_o \exp(-h/H_S) \quad (6.17)$$

$$n = N + 1 \quad (6.18)$$

The equations are integrated from $a = \text{zero}$ to $a = \text{measured range}$.

All that remains is to obtain expressions for ρ and E . In figure 6-1

$$R_i = R_o + h_i$$

$$R_f = R_o + h_f$$

$$T_1 = R_f \cos \theta_f - R_i$$

$$T_2 = R_f \sin \theta_f$$

$$! \rho = \sqrt{T_1^2 + T_2^2} ! \quad (6.19)$$

$$! \quad ! \quad ! \quad ! \quad ! \quad E = \arctan(T_1/T_2) ! \quad (6.20)$$

To integrate the previous equations, a fourth order Runge-Kutta-Gill integrator has been found to be very sufficient for most purposes. The Runge-Kutta-Gill integrator allows a maximum integration step size of about 10 000 meters for even the most precise work.

Let the state vector \underline{x} be defined by

$$\underline{x} = \begin{bmatrix} h \\ \theta \\ E_M \end{bmatrix} \quad (6.21)$$

Let

$$\underline{f}(\underline{x}) = \begin{bmatrix} dh/da \\ d\theta/da \\ dE_M/da \end{bmatrix} \quad (6.22)$$

Reference 3 gives the equations for many different integrators, along with their speed and accuracy. The equations for a fourth order Runge-Kutta integrator are

$$\underline{x}_n = \underline{x}$$

$$\rho_M = \rho_M + \Delta a$$

$$\underline{k}_1 = \Delta a \underline{f}(\underline{x})$$

$$\underline{x} = \underline{x}_n + a_1 \underline{k}_1$$

$$\underline{k}_2 = \Delta a \underline{f}(\underline{x})$$

$$\underline{x} = \underline{x}_n + b_1 \underline{k}_1 + b_2 \underline{k}_2$$

$$\underline{k}_3 = \Delta a \underline{f}(\underline{x})$$

$$\underline{x} = \underline{x}_n + c_1 \underline{k}_1 + c_2 \underline{k}_2 + c_3 \underline{k}_3$$

$$\underline{k}_4 = \Delta a \underline{f}(\underline{x})$$

$$\underline{x} = \underline{x}_n + d_1 \underline{k}_1 + d_2 \underline{k}_2 + d_3 \underline{k}_3 + d_4 \underline{k}_4$$

The Runge-Kutta-Gill constants (ref. 3) are

$$a_1 = 1/2 \quad b_1 = (\sqrt{2} - 1)/2 \quad b_2 = (2 - \sqrt{2})/2$$

$$c_1 = 0 \quad c_2 = -\sqrt{2}/2 \quad c_3 = (2 + \sqrt{2})/2$$

$$d_1 = 1/6 \quad d_2 = (2 - \sqrt{2})/6 \quad d_3 = (2 + \sqrt{2})/6 \quad d_4 = 1/6$$

The equations presented here are naturally integrated to a given value of measured range. To integrate to a desired altitude h_D , iterate with

$$\Delta a = (h_D - h)/\sin E_M \quad (6.23)$$

7.0 SLAB ATMOSPHERE APPROXIMATIONS

Figure 7-1 shows how an electromagnetic wave is refracted by a spherical slab atmosphere.

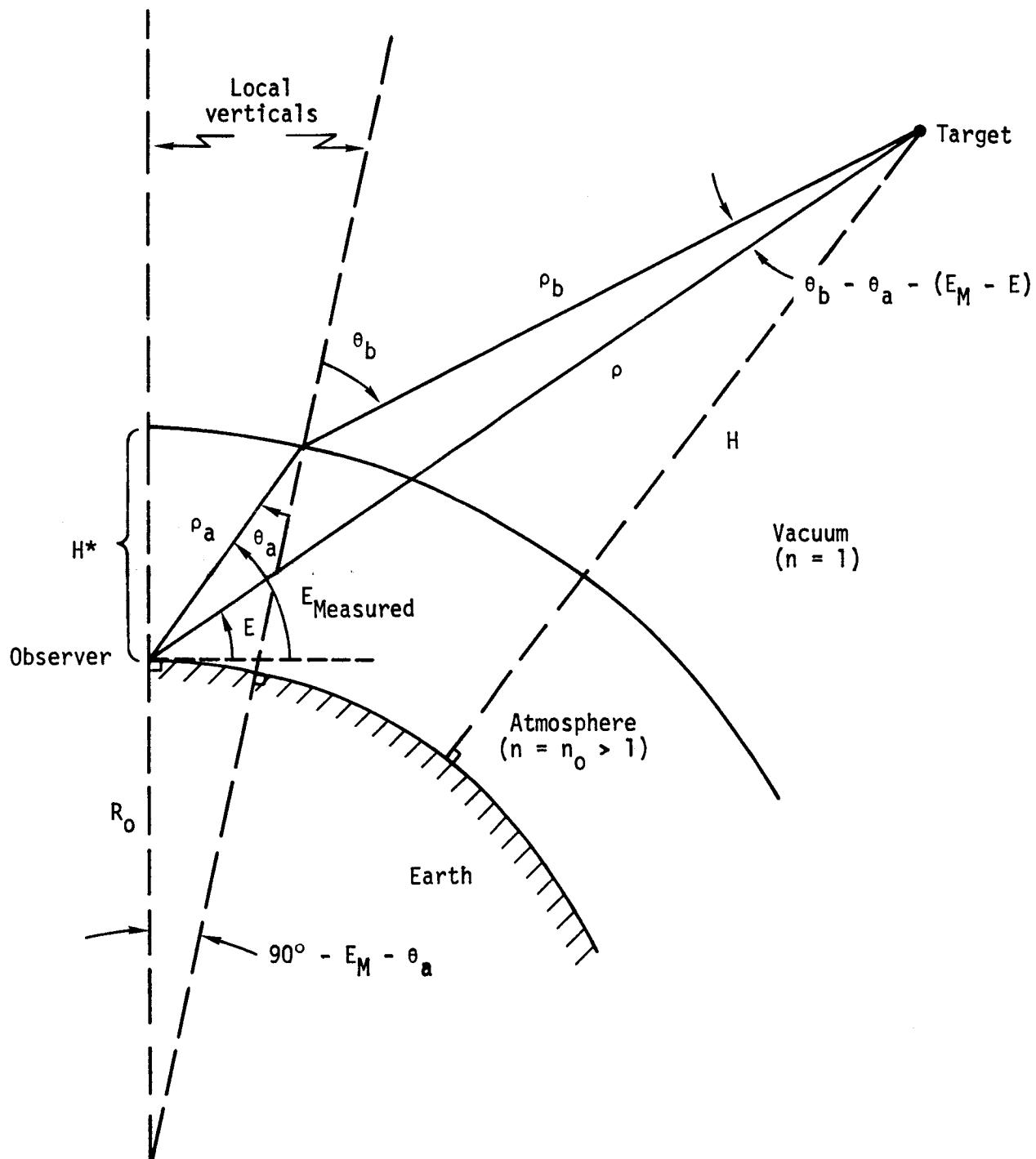


Figure 7-1.- Refraction by a spherical slab atmosphere.

The slab height H^* is chosen in the following manner. Let $E_M = E = 90$ degrees. Then the measured range is given by

$$\rho_M = n_0 H^* + (H - H^*) = \int_0^H n dh \quad (7.1)$$

Assuming an exponential atmosphere,

$$n = 1 + N_0 \exp(-h/H_S)$$

Substituting into equation (7.1), integrating, and solving for H^* gives

$$! \quad ! \quad H^* = (1 - \exp(-H/H_S)) H_S \quad ! \quad ! \quad (7.2)$$

where

$$! \quad ! \quad H = \sqrt{R_o^2 + \rho^2 + 2R_o\rho \sin E} - R_o \quad ! \quad ! \quad (7.3)$$

H is the altitude above the observer, who is at the radius R_o .

In figure 7-1, from the $R_o \rho_a$ triangle and the law of sines

$$\frac{R_o}{\sin \theta_a} = \frac{R_o + H^*}{\sin(90 + E_M)}$$

or

$$\sin \theta_a = \frac{1}{1 + H^*/R_o} \cos E_M \quad (7.4)$$

Also, this triangle and the law of cosines show that

$$\rho_a = R_o \left[\sqrt{(1 + H^*/R_o)^2 - \cos^2 E_M} - \sin E_M \right] \quad (7.5)$$

If n_o is the index of refraction of the slab atmosphere, then from Snell's law of refraction

$$\sin \theta_b = n_o \sin \theta_a$$

or, from equation (7.4)

$$\sin \theta_b = \frac{n_o}{1 + H^*/R_o} \cos E_M \quad (7.6)$$

From the ρ_a , ρ_b , ρ triangle and the law of sines,

$$\frac{\rho_a}{\sin(\theta_b - \theta_a - (E_M - E))} = \frac{\rho}{\sin(\theta_a - \theta_b + 180)}$$

or

$$\rho \sin(\theta_b - \theta_a - (E_M - E)) = \rho_a \sin(\theta_b - \theta_a) \quad (7.7)$$

Also, this triangle and the law of cosines show that

$$\rho_b = \sqrt{\rho^2 + \rho_a^2 - 2\rho\rho_a \cos(E_M - E)} \quad (7.8)$$

Equations (7.4) through (7.8) represent five nonlinear equations in the five unknowns: E_M , θ_a , θ_b , ρ_a , and ρ_b . The slab atmosphere equations are summarized as follows. The inputs are ρ and E ; the pertinent outputs are ρ_M and E_M .

$$H = \sqrt{R_o^2 + \rho^2 + 2R_o \rho \sin E} - R_o \quad (7.9)$$

$$H^* = (1 - \text{EXP}(-H/H_S)) H_S \quad (7.10)$$

$$E_M = E$$

DO a I = 1,10

$$\theta_a = \arcsin \frac{\cos E_M}{1 + H^*/R_o} \quad (7.11)$$

$$\theta_b = \arcsin \frac{n_o \cos E_M}{1 + H^*/R_o} \quad (7.12)$$

$$\rho_a = R_o (\sqrt{\sin^2 E_M + (H^*/R_o) (2 + H^*/R_o)} - \sin E_M) \quad (7.13)$$

$$a E_M = E + (\theta_b - \theta_a) - \arcsin \left[\frac{\rho_a}{\rho} \sin (\theta_b - \theta_a) \right] \quad (7.14)$$

$$\rho_b = \sqrt{\rho^2 + \rho_a^2 - 2\rho\rho_a \cos (E_M - E)} \quad (7.15)$$

$$\rho_M = n_o \rho_a + \rho_b \quad (7.16)$$

It should be noted that E_M in equations (7.9) through (7.16) is denoted by E_{Mi} elsewhere in this document. These equations are only good for positive values of altitude H above the observer.

The slab atmosphere equations (eqs. (7.9) through (7.16)) are time consuming to evaluate because of the 10-cycle iteration, and they are not very accurate. However, they do provide the basis for various approximations that are rapidly evaluated and can be made fairly accurate by empirical modifications.

Approximations of the refraction corrections

$$\Delta\rho = \rho_M - \rho$$

$$\Delta E = E_M - E$$

will now be obtained. Figure 7-1 shows that

$$\rho_b \approx \rho - \rho_a$$

Thus,

$$\Delta\rho = n_o \rho_a + \rho_b - \rho = (n_o - 1) \rho_a = N_o \rho_a$$

Assuming $E_M = E$ and $H^*/R_o \ll 2$ in equation (7.13) for ρ_a , yields

$$\Delta\rho_2 = N_o R_o \left[\sqrt{\sin^2 E + 2H^*/R_o} - \sin E \right] \quad (7.17)$$

or

$$\boxed{! \Delta\rho_2 = \frac{2N_o H^*}{\sqrt{\sin^2 E + 2H^*/R_o} + \sin E} !} \quad (7.18)$$

The subscript 2 denotes the second approximation studied.

The maximum value of $2H^*/R_o$ is about 0.002. So for the large value of E , $\Delta\rho_2$ is approximately

$$\boxed{! \Delta\rho_4 = \frac{N_o H^*}{\sin E} !} \quad (7.19)$$

$\Delta\rho_3$ is an empirical modification of $\Delta\rho_2$.

$$\Delta\rho_3 = \Delta\rho_2 \left[1 - 2.7 \cdot 10^7 N_o^{1.5} \frac{H^*}{R_o} (\cos E) 1.4 \cdot 10^6 N_o \right] \quad (7.20)$$

$\Delta\rho_3$ has an overall accuracy of about 1.6 percent. That is,

$$\sigma_{\Delta\rho_3} = 0.016 \Delta\rho_3 \quad (7.21)$$

Tables 7-1 through 7-12 show $\Delta\rho_2$, $\Delta\rho_3$, and $\Delta\rho_4$ compared with the precise values of $\rho_M - \rho$ obtained from the appendixes. $\Delta\rho_3$ is the most accurate approximation.

The approximate refraction correction for elevation angle can be obtained by observing that $\theta_b - \theta_a$ is a small angle. Thus, from equation (7.14)

$$\Delta E = E_M - E = \left(1 - \frac{\rho_a}{\rho} \right) (\theta_b - \theta_a) \quad (7.22)$$

Let

$$A = \frac{\cos E_M}{1 + H^*/R_o} \quad (7.23)$$

$$B = n_o A \quad (7.24)$$

Then from equations (7.11) and (7.12)

$$\begin{aligned}
 \theta_b - \theta_a &= \arcsin(A) - \arcsin(B) \\
 &= \arcsin \left[B\sqrt{1-A^2} - A\sqrt{1-B^2} \right] \\
 &\approx B\sqrt{1-A^2} - A\sqrt{1-B^2} \\
 &= \frac{B^2 - A^2}{B\sqrt{1-A^2} + A\sqrt{1-B^2}}
 \end{aligned}$$

Note again that $B = n_o A \approx A$. So,

$$\begin{aligned}
 \theta_b - \theta_a &= \frac{n_o^2 A^2 - A^2}{2A\sqrt{1-A^2}} \\
 &= \frac{(n_o^2 - 1) A}{2\sqrt{1-A^2}} \\
 &\approx \frac{N_o A}{\sqrt{1-A^2}}
 \end{aligned}$$

where

$$A = \frac{\cos E_M}{1 + H^*/R_o}$$

Thus $\theta_b - \theta_a$ is given approximately by

$$\theta_b - \theta_a = \frac{N_o \cos E_M}{\sqrt{\sin^2 E_M + 2H^*/R_o}} \quad (7.25)$$

Substituting equation (7.25) for $\theta_b - \theta_a$ and equation (7.13) for ρ_a into equation (7.22) and assuming $E_M = E$, yields

$$\Delta E_2 = \frac{N_o \cos E}{\sqrt{\sin^2 E + 2 H^*/R_o}} \left[1 - \frac{R_o}{\rho} (\sqrt{\sin^2 E + 2 H^*/R_o} - \sin E) \right] \quad (7.26)$$

or

$$\Delta E_2 = \frac{N_o \cos E}{\sqrt{\sin^2 E + 2 H^*/R_o}} \left[1 - \frac{2H^*/\rho}{\sqrt{\sin^2 E + 2 H^*/R_o} + \sin E} \right] \quad (7.27)$$

Tables 7-13 through 7-24 show that ΔE_2 is too small. Thus, the thought occurs to make the substitution

$$\frac{1}{\sqrt{\sin^2 E + 2 H^*/R_o}} = \frac{2}{\sqrt{\sin^2 E + 2 H^*/R_o} + \sin E}$$

Since $2H^*/R_o$ is on the order of 0.002, both expressions give $1/\sin E$ for the higher elevation angles. The one on the right will be larger for small values of E , which is exactly what is needed. Thus

$$\Delta E_3 = \frac{2N_o \cos E}{\sqrt{\sin^2 E + 2 H^*/R_o} + \sin E} \left[1 - \frac{2H^*/\rho}{\sqrt{\sin^2 E + 2 H^*/R_o} + \sin E} \right] \quad (7.28)$$

From equation (7.18)

$$\Delta \rho_2 = \frac{2N_o H^*}{\sqrt{\sin^2 E + 2 H^*/R_o} + \sin E}$$

Thus ΔE_3 could be rewritten as

$$\Delta E_3 = \frac{\Delta \rho_2 \cos E}{N_o H^*} (N_o - \Delta \rho_2 / \rho) \quad (7.29)$$

But $\Delta\rho_3$ was a better approximation than $\Delta\rho_2$. Thus

$$\boxed{\Delta E_4 = \frac{\Delta\rho_3 \cos E}{N_o H^*} (N_o - \Delta\rho_3/\rho)} \quad (7.30)$$

Tables 7-13 through 7-24 show that this is a better overall approximation than either ΔE_2 or ΔE_3 . However, at orbital altitudes for $E > 3$ degrees, ΔE_2 is best with practically no error for $E > 5$ degrees.

ΔE_{10} is obtained by assuming $2H^*/R_o \ll \sin^2 E$. From equation (7.27) for ΔE_2 ,

$$\boxed{\Delta E_{10} = \frac{N_o \cos E}{\sin E} \left[1 - \frac{H^*/\rho}{\sin E} \right]} \quad (7.31)$$

This approximation (tables 7-13 through 7-24) is only good for error analysis purposes.

The best overall approximation to ΔE is ΔE_{16} , which is given by the following equations. If $H > 10^7$ meters, set $H = 10^7$ meters:

$$A_1 = 0.394 + 1.16E5 N_o^2$$

$$A_2 = 0.009 + 7.1E5 N_o^2$$

$$A_3 = 0.004 + 1.E5 N_o^2$$

$$B_1 = 59.9 + 1.14E4 N_o - 1.9E7 N_o^2$$

$$B_2 = -3.379 - 1000 N_o + 8.E6 N_o^2$$

$$B_3 = 0.007$$

$$C_1 = 0.7181 - 246 N_o + 2.1E4 N_o^2$$

$$C_2 = 27.5 - 7800 N_o - 9.96E7 N_o^2$$

$$C_3 = -4.2 - 2300 N_o - 4.32E7 N_o^2$$

$$C_4 = 141.4 - 1.1E4 N_O + 4.03E8 N_O^2$$

$$C_5 = -20.4 - 9.E4 N_O - 3.1E7 N_O^2$$

$$A = \frac{A_1 + A_2 (H/R_O)}{1 + A_3 (H/R_O)}$$

$$B = \frac{B_1 + B_2 (H/R_O)}{1 + B_3 (H/R_O)}$$

$$C = \frac{C_1 + C_2 (H/R_O) + C_3 (H/R_O)^2}{1 + C_4 (H/R_O) + C_5 (H/R_O)^2}$$

$$\Delta E_{16} = \frac{N_O \cos E (1 + A \exp (-B \sin E))}{(1 - C) \sqrt{\sin^2 E + 2H^*/R_O} + C \sin E} \\ \times \left[1 - \frac{2H^*/\rho}{\sqrt{\sin^2 E + 2H^*/R_O} + \sin E} \right] \quad (7.32)$$

The empirical constants in the above equations were obtained by making a percentage least-squares fit using

$$N_O = 0.000255, 0.000325, 0.000395$$

$$H_S = 7892, 6735, 5446 \text{ meters}$$

$$E_M = 0, 0.5, 1, 2, 3, 4, 5 \text{ degrees}$$

$$H = 1.E3, 5.E3, 1.E4, 5.E4, 1.E5, 5.E5, 1.E6, 5.E6, 1.E7 \text{ meters}$$

There were 189 nonlinear equations in 33 unknowns. The fit was made relatively easy by using the Program SEARCH described in reference 4. The program also automatically determined the number of significant figures needed for the empirical coefficients. The fit accuracy was 1.70 percent, or

$$\sigma_{\Delta E_{16}} = 0.0170 \Delta E_{16} \quad (7.33)$$

The maximum fit error was 4.8 percent.

None of the above approximations for ΔE work sufficiently for low altitudes and short ranges; and in particular, the $\Delta \rho$ approximations do not work for negative altitudes. Therefore the short-range, low-altitude approximations were developed. $\Delta \rho_2$ was

$$\Delta \rho_2 = \frac{2N_o H^*}{\sqrt{\sin^2 E + 2H^*/R_o} + \sin E}$$

where

$$H^* = (1 - \exp(-H/H_S))H_S$$

Assuming H/H_S small and expanding the exponential gives

$$H^* = H(1 - .5 H/H_S)$$

Altitude is given by

$$H = \sqrt{R_o^2 + \rho^2 + 2R_o \rho \sin E} - R_o$$

Solving for $\sin E$,

$$\sin E = \frac{1}{2} \frac{R_o}{\rho} \left(\frac{H^2}{R_o^2} + 2 \frac{H}{R_o} - \frac{\rho^2}{R_o^2} \right)$$

But $H^2/R_o^2 \ll |2H/R_o|$, so

$$\sin E = \frac{H}{\rho} - \frac{1}{2} \frac{\rho}{R_o}$$

Now let $H^* = H$ and

$$\sin^2 E + 2H^*/R_o = \frac{H^2}{\rho^2} + \frac{1}{4} \frac{\rho^2}{R_o^2} - \frac{H}{R_o} + 2 \frac{H}{R_o} = \left(\frac{H}{\rho} + \frac{1}{2} \frac{\rho}{R_o} \right)^2$$

Thus

$$\sqrt{\sin^2 E + 2H^*/R_O} + \sin E = \frac{H}{\rho} + \frac{1}{2} \frac{\rho}{R_O} + \frac{H}{\rho} - \frac{1}{2} \frac{\rho}{R_O} = 2 \frac{H}{\rho}$$

Thus $\Delta\rho_2$ becomes

$$\Delta\rho_2 = \frac{2 N_O H (1 - 0.5 H/H_S)}{2H/\rho}$$

or

$$\Delta\rho_2 = N_O \rho (1 - 0.5 H/H_S) \quad (7.34)$$

ΔE_3 was

$$\Delta E_3 = \frac{\Delta\rho_2 \cos E}{N_O H^*} (N_O - \Delta\rho_2/\rho)$$

Substituting equation (7.34) gives, for H small

$$\Delta E_3 = \frac{N_O \rho \cos E}{2 H_S} (1 - 0.5H/H_S) \quad (7.35)$$

Equations (7.34) and (7.35) were empirically modified to give the following low-altitude, short-range corrections.

$$\Delta\rho_8 = N_O \rho \left(1 + 67 \frac{\rho^2}{R_O^2} \right) \left[1 - 0.49939 \frac{H}{H_S} + 0.17472 \left(\frac{H}{H_S} \right)^2 - 0.04344 \left(\frac{H}{H_S} \right)^3 \right] \quad (7.36)$$

$$\Delta E_{19} = \frac{N_O \rho \cos E}{2H_S} \left(1 + 46 \frac{\rho^2}{R_O^2} \right) \left[1 - 0.33324 \frac{H}{H_S} + 0.08558 \left(\frac{H}{H_S} \right)^2 - 0.01681 \left(\frac{H}{H_S} \right)^3 \right] \quad (7.37)$$

Tables 7-25 through 7-39 illustrate the accuracy of these approximations. They are accurate for $H = \pm 6000$ meters out to $\rho = 140\ 000$ meters. Thus, since the previous refraction corrections are unstable at short ranges and low altitudes, $\Delta\rho_8$ and ΔE_{19} should be used when $\rho < 140\ 000$ meters and $H < 6000$ meters. To illustrate the point further, table 7-40 shows $\Delta\rho_3$ and ΔE_4 for a short-range, low-altitude situation and neither perform well. $\Delta\rho_3$ and ΔE_4 are being used in a groundtracking program for the Space Shuttle. In this program the following logic is used to avoid disastrous results. If

$$(\rho < 0.02R_O \text{ AND } E < 0.6^\circ) \quad E = 0.6^\circ$$

As shown, ΔE_{16} was a successful empirical modification to obtain the elevation angle refraction correction. This prompted a further study of this type of empirical correction. The following equations were used to obtain the refraction corrections for

$$E_M = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1, 2, 3, 4, 5, 7, 10 \text{ degrees}$$

for various altitudes and various values of N_O and H_S .

$$\Delta\rho = \frac{N_O H^* (1 - A_p \exp(-B_p \sin E))}{(1 - C_p) \sqrt{\sin^2 E + 2H^*/R_O} + C_p \sin E} \quad (7.38)$$

$$\Delta E = \frac{N_o \cos E (1 - A_E \exp(-B_E \sin E))}{(1 - C_E) \sqrt{\sin^2 E + 2H^*/R_o} + C_E \sin E}$$

$$x \left[1 - \frac{2H^*/\rho}{\sqrt{\sin^2 E + 2H^*/R_o} + \sin E} \right] \quad (7.39)$$

A percentage least-squares fit was made using the Program SEARCH (ref. 4). Tables 7-41, 7-42, and 7-43 show the values of the empirical coefficients: $\Delta\rho$, B_ρ , C_ρ , A_E , B_E , and C_E . Also shown is the RMS percentage error of the fit. Note that B_ρ and B_E were limited to a maximum value of 200 to avoid numerical underflow in the computer. Due to the nonlinear equations, the least-squares fits were not unique. The possibility exists for other good solutions.

Tables 7-41, 7-42, and 7-61 illustrate the possibility of achieving significantly more accurate results than those obtained previously. This prompted an attempt to obtain orbital refraction corrections, using the tables as a starting point. The new algorithms for $\Delta\rho_{10}$ and ΔE_{21} proved very successful and provide high precision results. $\Delta\rho_{10}$ and ΔE_{21} were optimized for

$$N_o = 0.000255, 0.000290, 0.000325, 0.000360, 0.000395$$

$$H_S = 7892, 7350, 6735, 6091, 5446 \text{ meters}$$

$$E_M = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1, 2, 3, 4, 5, 7, 10 \text{ degrees}$$

$$H = 1.E5, 2.E5, 5.E5, 1.E6, 2.E6, 5.E6, 1.E7, 2.E7, 5.E7, 1.E8 \text{ meters}$$

There were a total of 700 nonlinear equations in 19 unknowns for $\Delta\rho_{10}$ and 21 unknowns for ΔE_{21} . The Program SEARCH was used to make a percentage least-squares fit of the data.

$\Delta\rho_{10}$ had an RMS percentage error of 0.375 percent for 700 points. The maximum error was 0.80 percent at $N_o = 0.000395$, $H = 2.E7$ meters and $E_M = 10$ degrees. The maximum absolute error in the data presented here was 0.9 meters at $N_o = 0.000395$, $H = 1.E5$ meters, and $E_M = 0$ degree.

ΔE_{21} had an RMS percentage error of 0.425 percent for 700 points. The maximum error was 1.84 percent at $N_o = 0.000395$, $H = 1.E5$ meters and $E_M = 0$ degrees. The next largest error was 1.05 percent at $N_o = 0.000360$, $H = 5.E6$ meters and $E_M = 10$ degrees. The maximum absolute error was 0.34 milliradian at $N_o = 0.000395$, $H = 1.E5$ meters and $E_M = 0$ degree.

The equations for $\Delta\rho_{10}$ are:

$$\begin{aligned}A_{\rho 1} &= 0.2753 - 167.6 N_o + 6.187E5 N_o^2 \\A_{\rho 2} &= 0.6653 - 3719.6 N_o + 5.823E6 N_o^2 \\A_{\rho 3} &= 2.3213 - 12741.7 N_o + 1.94943E7 N_o^2\end{aligned}$$

$$\begin{aligned}B_{\rho 1} &= 17.44 - 22770 N_o + 7.534E7 N_o^2 \\B_{\rho 2} &= 8.793 - 26270 N_o + 1.7931E7 N_o^2 \\B_{\rho 3} &= 0.6504 - 2276.2 N_o + 1.9666E6 N_o^2\end{aligned}$$

$$C_{\rho} = 0.5729$$

$$H = \sqrt{R_o^2 + \rho^2 + 2R_o \rho \sin E - R_o}$$

If (H. LT. 1.E5 meters) H = 1.E5 meters

If (H. GT. 1.E8 meters) H = 1.E8 meters

$$L_1 = \ln(H/R_o) + 4.156 \quad (R_o = 6378165 \text{ meters})$$

$$A_{\rho} = \frac{A_{\rho 1} + A_{\rho 2} L_1^2}{1 + A_{\rho 3} L_1^2} \quad B_{\rho} = \frac{B_{\rho 1} + B_{\rho 2} L_1^2}{1 + B_{\rho 3} L_1^2}$$

$$\Delta\rho_{10} = \frac{N_o H_S (1 - A_{\rho} \exp(-B_{\rho} \sin E))}{(1 - C_{\rho}) \sqrt{\sin^2 E + 2H_S/R_o} + C_{\rho} \sin E} \quad (7.40)$$

where $\Delta\rho_{10}$ has an RMS error of 0.37 percent.

The equations for ΔE_{21} are

$$\begin{aligned}A_E 1 &= -0.5579 + 2471.5 N_o - 3.6388 E6 N_o^2 \\A_E 2 &= 0.45795 - 2529.2 N_o + 5.2475 E6 N_o^2 \\A_E 3 &= 1.0658 - 5113.5 N_o + 1.0585 E7 N_o^2\end{aligned}$$

$$B_{E1} = 7.03 + 19.390 N_O + 3.636 E7 N_O^2$$

$$B_{E2} = 38.12 - 216.490 N_O + 5.2678 E8 N_O^2$$

$$B_{E3} = 1.8680 - 9014.1 N_O + 2.16403 E7 N_O^2$$

$$C_E = 0.5784$$

$$H = \sqrt{R_O^2 + \rho^2 + 2R_O \rho \sin E} - R_O$$

If (H. LT. 1.E5 meters) H = 1.E5 meters

If (H. GT. 1.E8 meters) H = 1.E8 meters

$$L_1 = \ln(H/R_O) + 4.156$$

$$L_2 = \ln(H/R_O) + 6.443$$

$$A_E = \frac{A_{E1} + A_{E2} L_2^2}{1 + A_{E3} L_2^2}$$

$$B_E = \frac{B_{E1} + B_{E2} L_1^2}{1 + B_{E3} L_1^2}$$

$$\begin{aligned} \Delta E_{21} &= \frac{N_O \cos E (1 - A_E \exp(-B_E \sin E))}{(1 - C_E) \sqrt{\sin^2 E + 2H_S/R_O} + C_E \sin E} \\ x &= \left[1 - \frac{2H_S/\rho}{\sqrt{\sin^2 E + 2H_S/R_O} + \sin E} \right] \end{aligned}$$
(7.41)

where ΔE_{21} has an RMS error of 0.43 percent.

Tables 7-44 through 7-49 illustrate the accuracy of $\Delta \rho_{10}$ and ΔE_{21} .

TABLE 7-1.- $\Delta\rho$ FOR $N_O = 0.000395$, $H_S = 5446$ METERS, $H = 10^2$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	48 649.47	-0.10074	19.12	19.15	19.09	--
.5	10 884.33	.47753	4.26	4.26	4.25	4.70
1	5 652.35	.98833	2.21	2.21	2.21	2.27
2	2 855.49	1.99411	1.12	1.12	1.12	1.12
3	1 907.79	2.99607	.75	.75	.75	.75
4	1 432.32	3.99705	.56	.56	.56	.56
5	1 146.74	4.99764	.45	.45	.45	.45
7	820.32	6.99832	.32	.32	.32	.32
10	575.80	9.99883	.23	.23	.23	.23
20	292.37	19.99943	.11	.11	.11	.11
30	200.00	29.99964	.08	.08	.08	.08
40	155.57	39.99975	.06	.06	.06	.06
50	130.54	49.99983	.05	.05	.05	.05
70	106.42	69.99992	.04	.04	.04	.04
90	100	90	.04	.04	.04	.04

TABLE 7-2.- $\Delta\rho$ FOR $N_O = 0.000395$, $H_S = 5446$ METERS, $H = 10^4$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	449 096.93	-0.74036	121.25	133.41	114.03	--
.5	362 042.56	-.04232	85.19	97.33	82.53	--
1	298 586.23	.57930	64.63	73.32	62.48	--
2	214 039.27	1.71787	42.46	46.19	40.71	60.32
3	162 802.03	2.79198	31.06	32.75	30.17	37.12
4	129 816.89	3.83683	24.29	25.15	24.04	27.02
5	107 326.43	4.86643	19.86	20.35	19.93	21.32
7	79 182.26	6.90264	14.50	14.69	14.65	15.05
10	56 572.62	9.93132	10.29	10.36	10.36	10.48
20	29 113.60	19.96640	5.27	5.28	5.28	5.30
30	19 965.99	29.97878	3.61	3.61	3.61	3.62
40	15 544.69	39.98539	2.81	2.81	2.81	2.81
50	13 048.85	49.98971	2.36	2.36	2.36	2.36
70	10 640.98	69.99553	1.92	1.92	1.92	1.92
90	10^4	90	1.81	1.81	1.81	1.81

TABLE 7-3.- $\Delta\rho$ FOR $N_O = 0.000395$, $H_S = 5446$ METERS, $H = 10^6$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	3 843 004.7	-1.1833	142.1	168.4	141.3	--
.5	3 751 457.8	-.3797	97.7	122.1	100.3	--
1	3 674 936.2	.3070	73.7	91.5	75.1	--
2	3 543 398.8	1.5221	48.6	56.8	48.4	81.0
3	3 426 768.0	2.6402	35.9	39.9	35.8	46.7
4	3 318 776.8	3.7136	28.3	30.4	28.7	33.2
5	3 216 948.1	4.7632	23.2	24.5	23.8	25.9
7	3 027 621.9	6.8253	17.1	17.6	17.5	18.1
10	2 772 989.0	9.8756	12.2	12.4	12.4	12.5
20	2 124 626.1	19.9386	6.3	6.3	6.3	6.3
30	1 703 701.9	29.9612	4.3	4.3	4.3	4.3
40	1 429 322.9	39.9732	3.3	3.3	3.3	3.3
50	1 248 558.0	49.9811	2.8	2.8	2.8	2.8
70	1 054 833.1	69.9918	2.3	2.3	2.3	2.3
90	10^6	90	2.2	2.2	2.2	2.2

TABLE 7-4.- $\Delta\rho$ FOR $N_0 = 0.000395$, $H_S = 5446$ METERS, $H = 10^8$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	106 325 018.1	-1.2411	144.1	172.1	144.8	--
.5	106 233 068.6	-.4157	98.5	124.0	101.9	--
1	106 155 336.3	.2825	74.1	92.4	75.8	--
2	106 018 961.0	1.5089	48.8	57.1	48.6	81.7
3	105 894 288.0	2.6321	35.9	39.9	35.9	46.8
4	105 775 082.7	3.7081	28.3	30.4	28.7	33.3
5	105 658 912.1	4.7592	23.3	24.5	23.8	25.9
7	105 431 752.9	6.8229	17.1	17.6	17.5	18.1
10	105 098 641.1	9.8743	12.2	12.4	12.4	12.5
20	104 034 062.9	19.9382	6.3	6.3	6.3	6.3
30	103 049 234.5	29.9609	4.3	4.3	4.3	4.3
40	102 168 301.8	39.9731	3.3	3.3	3.3	3.3
50	101 414 469.7	49.9810	2.8	2.8	2.8	2.8
70	100 362 576.6	69.9918	2.3	2.3	2.3	2.3
90	10^8	90	2.2	2.2	2.2	2.2

TABLE 7-5.- $\Delta\rho$ FOR $N_0 = 0.000325$, $H_S = 6735$ METERS, $H = 10^2$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	42 901.60	-0.05914	13.88	13.90	13.87	--
.5	10 740.41	.48523	3.47	3.47	3.46	3.81
1	5 631.08	.99226	1.81	1.82	1.81	1.86
2	2 852.70	1.99608	.92	.92	.92	.93
3	1 906.96	2.99738	.62	.62	.61	.62
4	1 431.97	3.99804	.46	.46	.46	.46
5	1 146.56	4.99843	.37	.37	.37	.37
7	820.26	6.99888	.26	.26	.26	.26
10	575.78	9.99922	.19	.19	.19	.19
20	292.37	19.99962	.09	.09	.09	.09
30	200.00	29.99976	.06	.06	.06	.06
40	155.57	39.99984	.05	.05	.05	.05
50	130.54	49.99988	.04	.04	.04	.04
70	106.42	69.99995	.03	.03	.03	.03
90	10^2	90	.03	.03	.03	.03

TABLE 7-6.- $\Delta\rho$ FOR $N_0 = 0.000325$, $H_S = 6735$ METERS, $H = 10^4$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	412 958.89	-0.46630	93.66	102.33	89.31	--
.5	341 993.38	.14058	70.79	78.85	68.67	--
1	286 650.64	.71288	55.89	61.87	54.15	136.07
2	209 203.16	1.80156	38.18	40.97	36.74	53.85
3	160 546.09	2.85168	28.41	29.74	27.55	34.03
4	128 634.54	3.88285	22.40	23.10	22.05	25.00
5	106 645.75	4.90372	18.41	18.81	18.35	19.81
7	78 904.77	6.92955	13.50	13.66	13.60	14.03
10	56 471.61	9.95019	9.61	9.67	9.67	9.80
20	29 100.92	19.97559	4.93	4.94	4.94	4.96
30	19 962.51	29.98458	3.38	3.38	3.38	3.39
40	15 543.40	39.98938	2.63	2.63	2.63	2.63
50	13 048.31	49.99252	2.21	2.21	2.21	2.21
70	10 640.89	69.99675	1.80	1.80	1.80	1.80
90	10^4	90	1.69	1.69	1.69	1.69

TABLE 7-7.- $\Delta\rho$ FOR $N_0 = 0.000325$, $H_S = 6735$ METERS, $H = 10^6$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	3 798 160.3	-0.7920	108.8	128.1	107.6	--
.5	3 722 846.2	-.1246	82.4	99.9	83.2	--
1	3 654 845.6	.4897	65.7	79.2	66.2	256.1
2	3 531 641.3	1.6330	45.9	53.0	45.7	76.8
3	3 418 864.5	2.7175	34.8	38.6	34.7	46.2
4	3 313 001.4	3.7721	27.8	30.0	28.1	33.3
5	3 212 492.3	4.8100	23.1	24.4	23.6	26.1
7	3 024 687.0	6.8585	17.1	17.7	17.6	18.3
10	2 771 181.2	9.8986	12.3	12.5	12.5	12.7
20	2 124 054.3	19.9497	6.4	6.4	6.4	6.4
30	1 703 468.0	29.9681	4.4	4.4	4.4	4.4
40	1 429 217.4	39.9780	3.4	3.4	3.4	3.4
50	1 248 508.9	49.9845	2.9	2.9	2.9	2.9
70	1 054 824.7	69.9933	2.3	2.3	2.3	2.3
90	10^6	90	2.2	2.2	2.2	2.2

TABLE 7-8.- $\Delta\rho$ FOR $N_O = 0.000325$, $H_S = 6735$ METERS, $H = 10^8$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	106 279 432.7	-0.8319	109.8	130.0	109.3	--
.5	106 203 715.4	-.1521	82.9	100.9	84.1	--
1	106 134 504.3	.4698	65.9	79.8	66.6	267.0
2	106 006 463.7	1.6214	46.0	53.2	45.8	77.4
3	105 885 647.6	2.7100	34.8	38.7	34.8	46.3
4	105 768 574.4	3.7670	27.8	30.0	28.1	33.3
5	105 653 728.3	4.8062	23.1	24.4	23.6	26.1
7	105 428 103.1	6.8562	17.2	17.7	17.6	18.3
10	105 096 144.9	9.8973	12.3	12.5	12.5	12.7
20	104 032 929.8	19.9492	6.4	6.4	6.4	6.4
30	103 048 583.8	29.9679	4.4	4.4	4.4	4.4
40	102 167 909.5	39.9779	3.4	3.4	3.4	3.4
50	101 414 239.7	49.9844	2.9	2.9	2.9	2.9
70	100 362 524.1	69.9932	2.3	2.3	2.3	2.3
90	10^8	90	2.2	2.2	2.2	2.2

TABLE 7-9.- $\Delta\rho$ FOR $N_0 = 0.000255$, $H_S = 7892$ METERS, $H = 10^2$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	40 072.35	-0.03701	10.18	10.19	10.17	--
.5	10 649.61	.49019	2.70	2.70	2.70	2.96
1	5 617.25	.99482	1.42	1.42	1.42	1.46
2	2 850.88	1.99737	.72	.72	.72	.73
3	1 906.42	2.99825	.48	.48	.48	.48
4	1 431.74	3.99868	.36	.36	.36	.36
5	1 146.44	4.99895	.29	.29	.29	.29
7	820.21	6.99925	.21	.21	.21	.21
10	575.76	9.99948	.15	.15	.15	.15
20	292.37	19.99975	.07	.07	.07	.07
30	200.00	29.99984	.05	.05	.05	.05
40	155.57	39.99989	.04	.04	.04	.04
50	130.54	49.99992	.03	.03	.03	.03
70	106.42	69.99997	.03	.03	.03	.03
90	10^2	90	.025	.025	.025	.025

TABLE 7-10.- $\Delta\rho$ FOR $N_O = 0.000225$, $H_S = 7892$ METERS, $H = 10^4$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	392 694.29	-0.30362	71.82	77.73	70.17	--
.5	329 555.51	.25972	56.21	61.60	55.60	--
1	278 774.64	.80478	45.36	49.44	44.77	--
2	205 784.19	1.86248	31.72	33.72	30.99	44.48
3	158 895.04	2.89626	23.87	24.85	23.31	28.61
4	127 752.64	3.91766	18.93	19.46	18.63	21.16
5	106 132.35	4.93214	15.61	15.91	15.50	16.81
7	78 693.05	6.95021	11.48	11.60	11.52	11.95
10	56 393.99	9.96473	8.19	8.23	8.23	8.35
20	29 091.13	19.98270	4.21	4.21	4.21	4.23
30	19 959.82	29.98906	2.89	2.89	2.89	2.89
40	15 542.41	39.99247	2.25	2.25	2.25	2.25
50	13 047.90	49.99470	1.89	1.89	1.89	1.89
70	10 640.83	69.99770	1.54	1.54	1.54	1.54
90	10^4	90	1.45	1.45	1.45	1.45

TABLE 7-11.- $\Delta\rho$ FOR $N_0 = 0.000255$, $H_S = 7892$ METERS, $H = 10^6$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	3 769 817.0	-0.5423	84.0	97.8	84.7	--
.5	3 702 453.3	.0584	66.8	79.3	68.5	--
1	3 639 361.1	.6311	54.9	65.0	56.3	--
2	3 521 682.4	1.7273	39.75	45.6	40.3	66.8
3	3 411 809.0	2.7866	30.7	34.1	31.0	41.4
4	3 307 679.8	3.8262	24.9	26.8	25.2	30.2
5	3 208 302.7	4.8541	20.8	22.0	21.2	23.8
7	3 021 863.4	6.8905	15.6	16.1	15.9	16.8
10	2 769 414.6	9.9211	11.2	11.4	11.4	11.7
20	2 123 487.6	19.9607	5.8	5.9	5.9	5.9
30	1 703 235.5	29.9750	4.0	4.0	4.0	4.0
40	1 429 112.3	39.9828	3.1	3.1	3.1	3.1
50	1 248 460.0	49.9879	2.6	2.6	2.6	2.6
70	1 054 816.4	69.9947	2.1	2.1	2.1	2.1
90	10^6	90	2.0	2.0	2.0	2.0

TABLE 7-12.- $\Delta\rho$ FOR $N_0 = 0.000255$, $H_S = 7892$ METERS, $H = 10^8$ METERS

E_M , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_2$, meters	$\Delta\rho_3$, meters	$\Delta\rho_4$, meters
0	106 250 348.0	-0.5708	84.5	98.7	85.5	--
.5	106 182 581.0	.0378	67.1	79.8	69.0	--
1	106 118 278.7	.6156	55.0	65.3	56.6	--
2	105 995 765.3	1.7177	39.8	45.7	40.4	67.1
3	105 877 855.4	2.7803	30.8	34.1	31.1	41.5
4	105 762 520.0	3.8217	24.9	26.9	25.2	30.2
5	105 648 811.0	4.8508	20.8	22.0	21.2	23.8
7	105 424 564.8	6.8884	15.6	16.1	15.9	16.8
10	105 093 690.1	9.9199	11.2	11.4	11.4	11.7
20	104 031 801.9	19.9602	5.9	5.9	5.9	5.9
30	103 047 934.4	29.9748	4.0	4.0	4.0	4.0
40	102 167 517.7	39.9826	3.1	3.1	3.1	3.1
50	101 414 010.0	49.9878	2.6	2.6	2.6	2.6
70	100 362 471.6	69.9947	2.1	2.1	2.1	2.1
90	10^8	90	2.0	2.0	2.0	2.0

TABLE 7-13.- ΔE FOR $N_0 = 0.000395$, $H_S = 5446$ METERS, $H = 10^2$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	48 649.47	-0.10074	1.758	0.217	0.620	1.249	--	1.660
.5	10 884.33	.47753	.392	.329	.360	.496	--	.459
1	5 652.35	.98833	.204	.194	.199	.265	--	.229
2	2 855.49	1.99411	.103	.102	.102	.128	.031	.107
3	1 907.79	2.99607	.069	.068	.069	.080	.048	.070
4	1 432.32	3.99705	.051	.051	.051	.056	.043	.052
5	1 146.74	4.99764	.041	.041	.041	.043	.037	.041
7	820.32	6.99832	.029	.029	.029	.030	.028	.029
10	575.80	9.99883	.020	.020	.020	.020	.020	.020
20	292.37	19.99943	.010	.010	.010	.010	.010	.010
30	200.00	29.99964	.006	.006	.006	.006	.006	.006
40	155.57	39.99975	.004	.004	.004	.004	.004	.004
50	130.54	49.99983	.003	.003	.003	.003	.003	.003
70	106.42	69.99992	.001	.001	.001	.001	.001	.001
90	100	90	0	0	0	0	0	0

TABLE 7-14.- ΔE FOR $N_0 = 0.000395$, $H_S = 5446$ METERS, $H = 10^4$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	449 096.93	-0.74036	12.922	2.447	7.226	8.897	--	13.092
.5	362 042.56	-.04232	9.465	3.329	6.791	7.624	--	9.443
1	298 586.23	.57930	7.343	3.811	6.059	6.418	--	7.280
2	214 039.27	1.71787	4.924	3.707	4.575	4.609	3.774	4.842
3	162 802.03	2.79198	3.631	3.137	3.507	3.494	3.424	3.567
4	129 816.89	3.83683	2.848	2.612	2.793	2.783	2.786	2.806
5	107 326.43	4.86643	2.331	2.203	2.303	2.299	2.307	2.305
7	79 182.26	6.90264	1.699	1.650	1.689	1.689	1.693	1.689
10	56 572.62	9.93132	1.199	1.182	1.196	1.196	1.198	1.195
20	29 113.60	19.96640	.586	.585	.586	.586	.587	.586
30	19 965.99	29.97878	.370	.370	.370	.370	.371	.370
40	15 544.69	39.98539	.255	.255	.255	.255	.255	.255
50	13 048.85	49.98971	.180	.180	.180	.180	.180	.180
70	10 640.98	69.99553	.078	.078	.078	.078	.078	.078
90	10 ⁴ 90	0	0	0	0	0	0	0

TABLE 7-15.- ΔE FOR $N_o = 0.000395$, $H_S = 5446$ METERS, $H = 10^6$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	3 843 004.7	-1.1833	20.65	7.60	27.49	23.53	---	20.78
.5	3 751 457.8	-3797	15.35	8.66	20.58	17.17	---	15.22
1	3 674 936.2	.3070	12.10	8.88	15.74	13.07	---	12.23
2	3 543 398.8	1.5221	8.34	7.71	10.01	8.57	14.01	8.59
3	3 426 768.0	2.6402	6.28	6.19	7.10	6.40	8.27	6.44
4	3 318 776.8	3.7136	5.00	5.01	5.44	5.14	5.93	5.10
5	3 216 948.1	4.7632	4.13	4.16	4.39	4.28	4.64	4.20
7	3 027 621.9	6.8253	3.05	3.07	3.16	3.15	3.25	3.08
10	2 772 989.0	9.8756	2.17	2.18	2.21	2.21	2.24	2.18
20	2 124 626.1	19.9386	1.07	1.07	1.08	1.08	1.08	1.07
30	1 703 701.9	29.9612	.68	.68	.68	.68	.68	.68
40	1 429 322.9	39.9732	.47	.47	.47	.47	.47	.47
50	1 248 558.0	49.9811	.33	.33	.33	.33	.33	.33
70	1 054 833.1	69.9918	.14	.14	.14	.14	.14	.14
90	10^6	90	0	0	0	0	0	0

TABLE 7-16.- ΔE FOR $N_O = 0.000395$, $H_S = 5446$ METERS, $H = 10^8$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{17} , mrad
0	106 325 018.1	-1.2411	21.66	8.43	31.47	26.48	---	22.35
.5	106 233 068.6	-.4157	15.98	9.39	22.70	18.66	---	16.02
1	106 155 336.3	.2825	12.52	9.47	16.93	13.90	---	12.69
2	106 018 961.0	1.5089	8.57	8.05	10.47	8.91	14.97	8.76
3	105 894 288.0	2.6321	6.42	6.38	7.32	6.58	8.58	6.51
4	105 775 082.7	3.7081	5.09	5.13	5.57	5.25	6.09	5.14
5	105 658 912.1	4.7592	4.20	4.24	4.48	4.36	4.74	4.23
7	105 431 752.9	6.8229	3.09	3.12	3.20	3.19	3.30	3.10
10	105 098 641.1	9.8743	2.19	2.21	2.24	2.24	2.27	2.20
20	104 034 062.9	19.9382	1.08	1.08	1.08	1.08	1.09	1.08
30	103 049 234.5	29.9609	.68	.68	.68	.68	.69	.68
40	102 168 301.8	39.9731	.47	.47	.47	.47	.47	.47
50	101 414 469.7	49.9810	.33	.33	.33	.33	.33	.33
70	100 362 576.6	69.9918	.14	.14	.14	.14	.14	.14
90	10 ⁸ 90 0	0	0	0	0	0	0	0

TABLE 7-17.- ΔE FOR $N_0 = 0.000325$, $H_S = 6735$ METERS, $H = 10^2$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	42 901.60	-0.05914	1.032	0.174	0.425	0.766	--	1.016
.5	10 740.41	.48523	.258	.217	.237	.320	--	.302
1	5 631.08	.99226	.135	.129	.132	.173	--	.152
2	2 852.70	1.99608	.068	.068	.068	.085	.010	.071
3	1 906.96	2.99738	.046	.046	.046	.054	.028	.046
4	1 431.97	3.99804	.034	.034	.034	.038	.027	.034
5	1 146.56	4.99843	.027	.027	.027	.029	.024	.027
7	820.26	6.99888	.020	.020	.020	.020	.018	.020
10	575.78	9.99922	.014	.014	.014	.014	.013	.014
20	292.37	19.99962	.007	.007	.007	.007	.007	.007
30	200.00	29.99976	.004	.004	.004	.004	.004	.004
40	155.57	39.99984	.003	.003	.003	.003	.003	.003
50	130.54	49.99988	.002	.002	.002	.002	.002	.002
70	106.42	69.99995	.001	.001	.001	.001	.001	.001
90	10 ²	90	0	0	0	0	0	0

TABLE 7-18.- ΔE FOR $N_0 = 0.000325$, $H_S = 6735$ METERS, $H = 10^4$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	412 958.89	-0.46630	8.138	1.873	4.666	5.736	--	8.210
.5	341 993.38	.14058	6.273	2.333	4.399	5.038	--	6.119
1	286 650.64	.71288	5.011	2.581	3.989	4.353	--	4.831
2	209 203.16	1.80156	3.463	2.521	3.124	3.240	2.149	3.334
3	160 546.09	2.85168	2.589	2.178	2.452	2.493	2.269	2.509
4	128 634.54	3.88285	2.045	1.840	1.980	1.996	1.925	1.997
5	106 645.75	4.90372	1.680	1.566	1.645	1.652	1.624	1.652
7	78 904.77	6.92955	1.230	1.185	1.217	1.217	1.211	1.218
10	56 471.61	9.95019	.869	.854	.865	.864	.865	.865
20	29 100.92	19.97559	.426	.424	.426	.426	.426	.426
30	19 962.51	29.98458	.269	.269	.269	.269	.269	.269
40	15 543.40	39.98938	.185	.185	.185	.185	.185	.185
50	13 048.31	49.99252	.131	.131	.131	.131	.131	.131
70	10 640.89	69.99675	.057	.057	.057	.057	.057	.057
90	10^4	90	0	0	0	0	0	0

TABLE 7-19.- ΔE FOR $N_0 = 0.000325$, $H_S = 6735$ METERS, $H = 10^6$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	3 798	160.3	-0.7920	13.82	6.07	17.05	14.59	---
.5	3 722	846.2	-1246	10.90	6.48	13.61	11.51	---
1	3 654	845.6	.4879	8.91	6.49	10.97	9.28	29.83
2	3 531	641.3	1.6330	6.41	5.73	7.51	6.51	10.64
3	3 418	864.5	2.7175	4.93	4.75	5.52	4.99	8.79
4	3 313	001.4	3.7721	3.98	3.93	4.32	4.06	6.56
5	3 212	492.3	4.8100	3.32	3.31	3.52	3.41	4.99
7	3 024	687.0	6.8585	2.47	2.48	2.56	2.55	4.03
10	2 771	181.2	9.8986	1.77	1.77	1.80	1.80	6.44
20	2 124	054.3	19.9497	.88	.88	.88	.88	1.78
30	1 703	468.0	29.9681	.56	.56	.56	.56	.56
40	1 429	217.4	39.9780	.38	.38	.38	.38	.38
50	1 248	508.9	49.9845	.27	.27	.27	.27	.27
70	1 054	824.7	69.9933	.12	.12	.12	.12	.12
90	106	90	0	0	0	0	0	0

TABLE 7-20.- ΔE FOR $N_0 = 0.000325$, $H_S = 6735$ METERS, $H = 10^8$ METERS

E_M , deg meters	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{17} , mrad
0	106 279 432.7	-0.8319	14.52	6.72	19.23	16.18	---	14.74
.5	106 203 715.4	-1521	11.38	7.04	14.94	12.46	---	11.34
1	106 134 504.3	.4698	9.25	6.95	11.82	9.88	39.33	9.24
2	106 006 463.7	1.6214	6.61	6.01	7.89	6.79	11.46	6.65
3	105 885 647.6	2.7100	5.06	4.92	5.73	5.15	6.86	5.10
4	105 768 574.4	3.7670	4.07	4.04	4.44	4.17	4.93	4.09
5	105 653 728.3	4.8062	3.38	3.39	3.61	3.49	3.86	3.40
7	105 428 103.1	6.8562	2.51	2.52	2.61	2.59	2.70	2.52
10	105 096 144.9	9.8973	1.79	1.80	1.83	1.83	1.86	1.80
20	104 032 929.8	19.9492	.89	.89	.89	.89	.90	.89
30	103 048 583.8	29.9679	.56	.56	.56	.56	.56	.56
40	102 167 909.5	39.9779	.39	.39	.39	.39	.39	.39
50	101 414 239.7	49.9844	.27	.27	.27	.27	.27	.27
70	100 362 524.1	69.9932	.12	.12	.12	.12	.12	.12
90	10^8	90	0	0	0	0	0	0

TABLE 7-21.- ΔE FOR $N_0 = 0.000255$, $H_S = 7892$ METERS, $H = 10^2$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	40 072.35	-0.03701	0.646	0.127	0.287	0.462	--	0.669
.5	10 649.61	.49019	.171	.145	.158	.203	--	.201
1	5 617.25	.99482	.090	.086	.088	.111	--	.101
2	2 850.88	1.99737	.046	.045	.046	.055	--	.048
3	1 906.42	2.99825	.031	.030	.031	.036	.017	.031
4	1 431.74	3.99868	.023	.023	.023	.026	.017	.023
5	1 146.44	4.99895	.018	.018	.018	.020	.015	.018
7	820.21	6.99925	.013	.013	.013	.013	.012	.013
10	575.76	9.99948	.009	.009	.009	.009	.009	.009
20	292.37	19.99975	.004	.004	.004	.004	.004	.004
30	200.00	29.99984	.003	.003	.003	.003	.003	.003
40	155.57	39.99989	.002	.002	.002	.002	.002	.002
50	130.54	49.99992	.001	.001	.001	.001	.001	.001
70	106.42	69.99997	.001	.001	.001	.001	.001	.001
90	10 ²	90	0	0	0	0	0	0

TABLE 7-22.- ΔE FOR $N_0 = 0.000255$, $H_S = 7892$ METERS, $H = 10^4$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	392 694.29	-0.30362	5.299	1.343	3.068	3.704	--	5.409
.5	329 555.51	.25972	4.194	1.606	2.901	3.319	--	4.083
1	278 774.64	.80478	3.407	1.747	2.656	2.923	--	3.259
2	205 784.19	1.86248	2.400	1.711	2.125	2.237	1.195	2.290
3	158 895.04	2.89626	1.811	1.496	1.693	1.744	1.481	1.744
4	127 752.64	3.91766	1.437	1.276	1.379	1.404	1.305	1.398
5	106 132.35	4.93214	1.184	1.093	1.152	1.164	1.119	1.160
7	78 693.05	6.95021	.869	.833	.857	.859	.846	.859
10	56 393.99	9.96473	.615	.603	.611	.612	.608	.612
20	29 091.13	19.98270	.302	.301	.302	.302	.301	.302
30	19 959.82	29.98906	.191	.191	.191	.191	.191	.191
40	15 542.41	39.99247	.131	.131	.131	.131	.131	.131
50	13 047.90	49.99470	.093	.093	.093	.093	.093	.093
70	10 640.83	69.99770	.040	.040	.040	.040	.040	.040
90	10 ⁴	90	0	0	0	0	0	0

TABLE 7-23 .- ΔE FOR $N_0 = 0.000255$, $H_S = 7892$ METERS, $H = 10^6$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	3 769 817.0	-0.5423	9.47	4.52	11.13	9.78	--	9.31
.5	3 702 453.3	.0584	7.71	4.69	9.20	8.05	--	7.49
1	3 639 361.1	.6311	6.44	4.65	7.65	6.70	--	6.29
2	3 521 682.4	1.7273	4.76	4.16	5.48	4.88	7.83	4.74
3	3 411 809.0	2.7866	3.72	3.52	4.14	3.79	4.99	3.74
4	3 307 679.8	3.8262	3.03	2.96	3.29	3.09	3.68	3.06
5	3 208 302.7	4.8541	2.55	2.52	2.71	2.61	2.92	2.57
7	3 021 863.4	6.8905	1.91	1.91	1.98	1.96	2.06	1.93
10	2 769 414.6	9.9211	1.38	1.38	1.41	1.40	1.43	1.39
20	2 123 487.6	19.9607	.69	.69	.69	.69	.69	.69
30	1 703 235.5	29.9750	.44	.44	.44	.44	.44	.44
40	1 429 112.3	39.9828	.30	.30	.30	.30	.30	.30
50	1 248 460.0	49.9879	.21	.21	.21	.21	.21	.21
70	1 054 816.4	69.9947	.09	.09	.09	.09	.09	.09
90	10^6	90	0	0	0	0	0	0

TABLE 7-24.- ΔE FOR $N_0 = 0.000255$, $H_S = 7892$ METERS, $H = 10^8$ METERS

E_M , deg	ρ , meters	E , deg	$E_M - E$, mrad	ΔE_2 , mrad	ΔE_3 , mrad	ΔE_4 , mrad	ΔE_{10} , mrad	ΔE_{16} , mrad
0	106 250	348.0	-0.5708	9.96	5.01	12.46	10.80	—
.5	106 182	581.0	.0378	8.07	5.11	10.09	8.72	—
1	106 118	278.7	.6156	6.71	5.00	8.25	7.16	—
2	105 995	765.3	1.7177	4.93	4.38	5.78	5.11	8.48
3	105 877	855.4	2.7803	3.83	3.66	4.31	3.93	5.24
4	105 762	520.0	3.8217	3.11	3.06	3.39	3.18	3.81
5	105 648	811.0	4.8508	2.60	2.59	2.78	2.67	3.00
7	105 424	564.8	6.8884	1.95	1.95	2.03	2.01	2.11
10	105 093	690.1	9.9199	1.40	1.40	1.43	1.43	1.40
20	104 031	801.9	19.9602	.69	.69	.70	.70	.70
30	103 047	934.4	29.9748	.44	.44	.44	.44	.44
40	102 167	5117.7	39.9826	.30	.30	.30	.30	.30
50	101 414	010.0	49.9878	.21	.21	.21	.21	.21
70	100 362	471.6	69.9947	.09	.09	.09	.09	.09
90		108	90	0	0	0	0	0

TABLE 7-25.- $\Delta\varphi_8$ AND ΔE_{19} FOR $N_O = 0.000255$, $H_S = 7892$ METERS, $\rho_M = 20,000$ METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\varphi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
28	9408.9	19 997.0	27.9886	2.98	2.96	0.200	0.199
24	8155.6	19 996.8	23.9876	3.18	3.18	.216	.215
20	6862.5	19 996.6	19.9867	3.41	3.42	.232	.232
16	5535.9	19 996.3	15.9857	3.67	3.68	.249	.249
12	4182.2	19 996.0	11.9847	3.96	3.97	.267	.267
8	2807.9	19 995.7	7.9837	4.29	4.30	.285	.285
4	1419.9	19 995.3	3.9826	4.67	4.67	.304	.304
0	24.9	19 994.9	-.0185	5.09	5.09	.323	.323
-4	-1370.4	19 994.4	-.0196	5.57	5.57	.342	.342
-8	-2759.0	19 993.9	-.0207	6.11	6.11	.361	.361
-12	-4134.3	19 993.3	-.0217	6.70	6.71	.379	.379
-16	-5489.4	19 992.6	-.0228	7.37	7.38	.397	.397
-20	-6817.7	19 991.9	-.0237	8.10	8.11	.414	.414
-24	-8112.8	19 991.1	-.0246	8.91	8.90	.429	.428
-28	-9368.2	19 990.2	-.0253	9.78	9.75	.441	.440

TABLE 7-26.- $\Delta\varphi_8$ AND ΔE_{19} FOR $N_o = 0.000255$, $H_S = 7892$ METERS, $\rho_M = 40$ 000 METERS

E_M , deg	H, meters	ρ , meters	E, deg	$\rho_M - \rho$, meters	$\Delta\varphi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
10	7047.2	39 993.2	9.9723	6.76	6.77	0.483	0.484
8	5668.5	39 992.7	7.9707	7.29	7.31	.511	.512
6	4282.7	39 992.1	5.9690	7.89	7.91	.541	.542
4	2891.5	39 991.4	3.9671	8.56	8.57	.574	.574
2	1496.6	39 990.7	1.9652	9.31	9.32	.608	.608
0	99.6	39 989.8	-.0369	10.16	10.16	.645	.645
-2	-1297.7	39 988.9	-2.0392	11.11	11.11	.684	.684
-4	-2693.6	39 987.8	-4.0416	12.18	12.19	.726	.726
-6	-4086.4	39 986.6	-6.0442	13.39	13.41	.771	.771
-8	-5474.3	39 985.3	-8.0469	14.75	14.77	.819	.819
-10	-6855.7	39 983.7	-10.0498	16.28	16.30	.870	.870

TABLE 7-27.- $\Delta\rho_8$ AND ΔE_{19} FOR $N_O = 0.000255$, $H_S = 7892$ METERS, $\rho_M = 60\ 000$ METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
6	6504.7	59 989.5	5.9571	10.48	10.51	0.749	0.749
5	5461.5	59 988.9	4.9553	11.10	11.14	.780	.780
4	4416.5	59 988.2	3.9534	11.78	11.81	.813	.813
3	3369.9	59 987.5	2.9515	12.52	12.54	.847	.847
2	2322.2	59 986.7	1.9493	13.32	13.34	.884	.884
1	1273.5	59 985.8	.9471	14.20	14.21	.923	.923
0	224.2	59 984.8	-.0552	15.16	15.17	.964	.964
-1	-825.3	59 983.8	-1.0578	16.21	16.22	1.008	1.008
-2	-1874.8	59 982.6	-2.0604	17.36	17.37	1.055	1.054
-3	-2923.9	59 981.4	-3.0633	18.61	18.64	1.104	1.104
-4	-3972.4	59 980.0	-4.0663	19.98	20.02	1.157	1.156
-5	-5019.9	59 978.5	-5.0695	21.49	21.53	1.212	1.212
-6	-6066.0	59 976.9	-6.0729	23.13	23.18	1.272	1.272

TABLE 7-28.- $\Delta\phi_8$ AND ΔE_{19} FOR $N_O = 0.000255$, $H_S = 7892$ METERS, $\rho_M = 100$ 000 METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\phi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
3.2	6234.3	99 982.10	3.12686	17.90	17.96	1.277	1.275
2.8	5534.4	99 981.40	2.72490	18.60	18.68	1.311	1.310
2.4	4834.0	99 980.65	2.32285	19.35	19.43	1.346	1.345
2.0	4133.3	99 979.85	1.92073	20.15	20.22	1.384	1.382
1.6	3432.1	99 979.01	1.51851	20.99	21.05	1.422	1.420
1.2	2730.6	99 978.12	1.11621	21.88	21.93	1.462	1.460
.8	2028.7	99 977.18	.71380	22.82	22.87	1.504	1.502
.4	1326.6	99 976.18	.31130	23.82	23.86	1.548	1.546
0	624.2	99 975.12	-.09132	24.88	24.92	1.594	1.591
-.4	-78.5	99 974.00	-.49405	26.00	26.04	1.641	1.639
-.8	-781.4	99 972.80	-.89689	27.20	27.24	1.691	1.688
-1.2	-1484.5	99 971.54	-1.29987	28.46	28.51	1.743	1.740
-1.6	-2187.8	99 970.19	-1.70297	29.81	29.87	1.797	1.795
-2.0	-2891.3	99 968.76	-2.10621	31.24	31.32	1.854	1.852
-2.4	-3594.9	99 967.25	-2.50960	32.75	32.85	1.913	1.911
-2.8	-4298.5	99 965.63	-2.91315	34.37	34.48	1.975	1.973
-3.2	-5002.3	99 963.91	-3.31685	36.09	36.22	2.039	2.038
-3.6	-5706.1	99 962.08	-3.72073	37.92	38.06	2.107	2.106

TABLE 7-29.- $\Delta\phi_8$ AND ΔE_{19} FOR $N_O = 0.000255$, $H_S = 7892$ METERS, $\rho_M = 140,000$ METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\phi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
2.0	6163.9	139 974.48	1.89592	25.52	25.64	1.817	1.811
1.6	5178.3	139 973.04	1.49198	26.96	27.09	1.885	1.880
1.2	4191.8	139 971.48	1.08780	28.52	28.65	1.958	1.952
.8	3204.4	139 969.78	.68338	30.22	30.33	2.035	2.029
.4	2216.2	139 967.95	.27870	32.05	32.15	2.117	2.110
0	1227.1	139 965.96	-.12628	34.04	34.13	2.204	2.196
-.4	237.2	139 963.80	-.53157	36.20	36.30	2.296	2.288
-.8	-753.6	139 961.45	-.93719	38.55	38.66	2.394	2.386
-1.2	-1745.3	139 958.90	-1.34316	41.10	41.24	2.499	2.491
-1.6	-2737.9	139 956.11	-1.74952	43.89	44.06	2.610	2.603
-2.0	-3731.5	139 953.08	-2.15630	46.92	47.15	2.728	2.722
-2.4	-4725.9	139 949.77	-2.56353	50.23	50.51	2.854	2.849
-2.8	-5721.3	139 946.15	-2.97124	53.85	54.17	2.989	2.984

TABLE 7-30.- $\Delta\phi_8$ AND ΔE_{19} FOR $N_o = 0.000325$, $H_S = 6735$ METERS, $\rho_M = 20,000$ METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\phi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
28	9407.2	19 996.5	27.9839	3.50	3.41	0.281	0.279
24	8153.7	19 996.2	23.9824	3.77	3.74	.307	.305
20	6860.4	19 995.9	19.9809	4.08	4.08	.333	.332
16	5533.6	19 995.6	15.9794	4.44	4.44	.360	.359
12	4179.6	19 995.2	11.9778	4.85	4.86	.388	.388
8	2805.2	19 994.7	7.9760	5.32	5.33	.418	.418
4	1416.9	19 994.1	3.9743	5.86	5.87	.449	.449
0	21.7	19 993.5	-.0276	6.49	6.49	.482	.482
-4	-1373.7	19 992.8	-.4.0296	7.21	7.21	.516	.516
-8	-2762.5	19 992.0	-.8.0315	8.04	8.04	.550	.551
-12	-4137.8	19 991.0	-.12.0335	8.98	8.99	.585	.586
-16	-5492.9	19 990.0	-.16.0355	10.05	10.06	.619	.620
-20	-6821.1	19 988.7	-.20.0374	11.26	11.25	.652	.654
-24	-8115.9	19 987.4	-.24.0392	12.61	12.56	.683	.685
-28	-9371.0	19 985.9	-.28.0410	14.11	13.98	.716	.713

TABLE 7-31.- $\Delta\varphi_8$ AND ΔE_{19} FOR $N_o = 0.000325$, $H_S = 6735$ METERS, $\rho_M = 40$ 000 METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\varphi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
10	7038.8	39 991.9	9.9604	8.08	8.07	0.692	0.691
8	5659.4	39 991.2	7.9578	8.82	8.83	.737	.738
6	4272.8	39 990.4	5.9549	9.65	9.67	.787	.787
4	2880.7	39 989.4	3.9518	10.60	10.62	.840	.841
2	1484.8	39 988.3	1.9485	11.69	11.70	.899	.899
0	86.9	39 987.0	-.0551	12.95	12.95	.962	.962
-2	-1311.5	39 985.6	-2.0591	14.38	14.39	1.032	1.032
-4	-2708.6	39 984.0	-4.0634	16.04	16.05	1.107	1.108
-6	-4102.6	39 982.1	-6.0682	17.94	17.96	1.190	1.190
-8	-5491.8	39 979.9	-8.0733	20.13	20.15	1.280	1.280
-10	-6874.6	39 977.4	-10.0789	22.65	22.64	1.378	1.377

TABLE 7-32.- $\Delta\varphi_8$ AND ΔE_{19} FOR $N_o = 0.000325$, $H_S = 6735$ METERS, $\rho_M = 60$ 000 METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\varphi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
6	6485.0	59 987.4	5 9.9384	12.59	12.60	1.075	1.075
5	5440.6	59 986.5	4 9.9355	13.46	13.49	1.126	1.126
4	4394.3	59 985.6	3 9.9324	14.41	14.44	1.180	1.181
3	3346.3	59 984.5	2 9.9290	15.46	15.49	1.239	1.239
2	2297.0	59 983.4	1 9.9255	16.62	16.63	1.301	1.301
1	1246.7	59 982.1	.9.9216	17.90	17.91	1.368	1.367
0	195.7	59 980.7	-.0825	19.33	19.33	1.440	1.439
-1	-855.8	59 979.1	-1.0869	20.91	20.91	1.517	1.516
-2	-1907.4	59 977.3	-2.0917	22.66	22.68	1.600	1.600
-3	-2958.7	59 975.4	-3.0968	24.61	24.64	1.689	1.689
-4	-4009.6	59 973.2	-4.1023	26.78	26.83	1.785	1.786
-5	-5059.7	59 970.8	-5.1082	29.20	29.26	1.889	1.890
-6	-6108.7	59 968.1	-6.1146	31.90	31.94	2.001	2.001

TABLE 7-33.- $\Delta\phi_8$ AND ΔE_{19} FOR $N_O = 0.000325$, $H_S = 6735$ METERS, $\rho_M = 100,000$ METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\phi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
3.2	6177.9	99 978.36	3.09461	21.64	21.65	1.839	1.835
2.8	5475.7	99 977.38	2.69136	22.62	22.66	1.896	1.892
2.4	4773.0	99 976.32	2.28795	23.68	23.73	1.956	1.952
2.0	4069.7	99 975.20	1.88438	24.80	24.85	2.018	2.014
1.6	3365.9	99 973.99	1.48063	26.01	26.05	2.083	2.079
1.2	2661.5	99 972.70	1.07670	27.30	27.32	2.152	2.148
.8	1956.7	99 971.33	.67257	28.67	28.69	2.224	2.220
.4	1251.3	99 969.85	.26824	30.15	30.15	2.300	2.295
0	545.6	99 968.27	-.13632	31.73	31.73	2.379	2.375
-.4	-160.7	99 966.58	-.54112	33.42	33.42	2.463	2.459
-.8	-867.4	99 964.76	-.94616	35.24	35.25	2.551	2.547
-1.2	-1574.5	99 962.81	-1.35147	37.19	37.21	2.644	2.640
-1.6	-2282.1	99 960.72	-1.75707	39.28	39.33	2.741	2.739
-2.0	-2990.0	99 958.47	-2.16292	41.53	41.60	2.844	2.843
-2.4	-3698.4	99 956.04	-2.56917	43.96	44.05	2.953	2.952
-2.8	-4407.2	99 953.44	-2.97573	46.56	46.68	3.067	3.067
-3.2	-5116.4	99 950.64	-3.38264	49.36	49.50	3.188	3.189
-3.6	-5826.0	99 947.62	-3.78994	52.38	52.53	3.315	3.317

TABLE 7-34.- $\Delta\phi_8$ AND ΔE_{19} FOR $N_o = 0.000325$, $H_S = 6735$ METERS, $\rho_M = 140,000$ METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\phi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
2.0	6050.3	139 968.96	1.84947	31.04	31.03	2.627	2.614
1.6	5058.2	139 966.92	1.44288	33.08	33.11	2.742	2.729
1.2	4064.7	139 964.69	1.03584	35.31	35.34	2.865	2.852
.8	3069.7	139 962.24	.62829	37.76	37.78	2.997	2.983
.4	2073.2	139 959.54	.22021	40.46	40.45	3.138	3.124
0	1075.1	139 956.57	-.18846	43.43	43.41	3.289	3.275
-.4	75.3	139 953.29	-.59777	46.71	46.69	3.452	3.439
-.8	-926.2	139 949.67	-1.00779	50.33	50.34	3.627	3.615
-1.2	-1929.5	139 945.67	-1.41857	54.33	54.39	3.815	3.805
-1.6	-2934.8	139 941.24	-1.83018	58.76	58.89	4.017	4.011
-2.0	-3942.2	139 936.32	-2.24271	63.68	63.89	4.236	4.234
-2.4	-4951.9	139 930.86	-2.65625	69.14	69.43	4.472	4.474
-2.8	-5964.0	139 924.78	-3.07088	75.22	75.55	4.728	4.734

TABLE 7-35.- $\Delta\rho_8$ AND ΔE_{19} FOR $N_o = 0.000395$, $H_S = 5446$ METERS, $\rho_M = 20\ 000$ METERS

E_M , deg	H, meters	ρ , meters	E, deg	$\rho_M - \rho$, meters	$\Delta\rho_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
28	9405.2	19 996.2	27.9777	3.76	3.44	0.389	0.380
24	8151.4	19 995.9	23.9756	4.10	3.94	.426	.422
20	6857.7	19 995.5	19.9732	4.50	4.44	.467	.465
16	5530.5	19 995.0	15.9707	4.97	4.96	.512	.511
12	4176.1	19 994.5	11.9680	5.52	5.53	.559	.559
8	2801.3	19 993.8	7.9651	6.18	6.19	.609	.610
4	1412.6	19 993.0	3.9619	6.96	6.97	.665	.665
0	16.9	19 992.1	-.0415	7.89	7.89	.724	.725
-4	-1379.0	19 991.0	-.4 0452	8.99	8.99	.788	.789
-8	-2768.3	19 989.7	-.8 0491	10.30	10.31	.857	.857
-12	-4144.0	19 988.2	-.12 0532	11.84	11.85	.929	.929
-16	-5499.3	19 986.3	-.16 0575	13.65	13.64	1.004	1.004
-20	-6827.7	19 984.2	-.20 0620	15.77	15.69	1.082	1.080
-24	-8122.4	19 981.8	-.24 0665	18.23	17.99	1.160	1.154
-28	-9377.0	19 978.9	-.28 0709	21.07	20.53	1.237	1.224

TABLE 7-36.- $\Delta\phi_8$ AND ΔE_{19} FOR $N_o = 0.000395$, $H_S = 5446$ METERS, $\rho_M = 40,000$ METERS

E_M , deg	H, meters	ρ , meters	E, deg	$\rho_M - \rho$, meters	$\Delta\phi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
10	7027.6	39 991.1	9.9443	8.90	8.76	0.972	0.968
8	5646.9	39 990.1	7.9400	9.86	9.84	1.048	1.047
6	4258.9	39 989.0	5.9351	10.99	11.02	1.132	1.132
4	2865.2	39 987.7	3.9298	12.32	12.34	1.225	1.226
2	1467.5	39 986.1	1.9238	13.89	13.89	1.330	1.330
0	67.5	39 984.3	-.0829	15.74	15.74	1.446	1.447
-2	-1333.2	39 982.1	-2.0904	17.94	17.95	1.577	1.578
-4	-2732.9	39 979.4	-4.0988	20.56	20.58	1.724	1.725
-6	-4129.8	39 976.3	-6.1082	23.69	23.72	1.889	1.891
-8	-5522.3	39 972.6	-8.1189	27.41	27.41	2.074	2.075
-10	-6908.8	39 968.1	-10.1308	31.87	31.71	2.283	2.280

TABLE 7-37.- $\Delta\rho_8$ AND ΔE_{19} FOR No = 0.000395, $H_S = 5446$ METERS, $\rho_M = 60$ 000 METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\rho_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
6	6458.4	59 986.0	5.9129	13.98	13.85	1.520	1.515
5	5411.9	59 984.9	4.9080	15.13	15.10	1.605	1.604
4	4363.2	59 983.6	3.9027	16.42	16.44	1.699	1.698
3	3312.6	59 982.1	2.8969	17.88	17.90	1.800	1.800
2	2260.4	59 980.5	1.8905	19.53	19.54	1.911	1.911
1	1206.8	59 978.6	.8836	21.40	21.39	2.032	2.031
0	152.2	59 976.5	-.1240	23.52	23.50	2.164	2.164
-1	-903.2	59 974.1	-1.1323	25.93	25.92	2.309	2.309
-2	-1959.3	59 971.3	-2.1414	28.69	28.70	2.468	2.470
-3	-3015.6	59 968.2	-3.1514	31.83	31.87	2.643	2.646
-4	-4071.9	59 964.6	-4.1625	35.43	35.48	2.835	2.840
-5	-5128.1	59 960.5	-5.1746	39.55	39.58	3.047	3.052
-6	-6183.9	59 955.7	-6.1880	44.27	44.21	3.281	3.285

TABLE 7-38.- $\Delta\phi_8$ AND ΔE_{19} FOR $N_O = 0.000395$, $H_S = 5446$ METERS, $\rho_M = 100\ 000$ METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\phi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
3.2	6100.4	99 975.76	3.05018	24.24	24.04	2.615	2.601
2.8	5394.2	99 974.43	2.64463	25.57	25.47	2.712	2.701
2.4	4687.1	99 972.99	2.23874	27.01	26.97	2.814	2.806
2.0	3979.1	99 971.42	1.83251	28.58	28.56	2.923	2.916
1.6	3270.3	99 969.72	1.42590	30.28	30.25	3.039	3.032
1.2	2560.5	99 967.88	1.01887	32.12	32.08	3.161	3.154
.8	1849.8	99 965.87	.61141	34.13	34.07	3.292	3.285
.4	1138.3	99 963.69	.20347	36.31	36.24	3.430	3.424
0	425.7	99 961.31	-.20498	38.69	38.61	3.578	3.572
-.4	-287.9	99 958.71	-.61398	41.29	41.21	3.735	3.731
-.8	-1002.4	99 955.87	-1.02357	44.13	44.07	3.902	3.901
-1.2	-1718.1	99 952.77	-1.43381	47.23	47.21	4.081	4.083
-1.6	-2434.8	99 949.37	-1.84475	50.63	50.65	4.272	4.278
-2.0	-3152.8	99 945.65	-2.25644	54.35	54.42	4.476	4.486
-2.4	-3871.9	99 941.57	-2.66894	58.43	58.54	4.694	4.709
-2.8	-4592.4	99 937.09	-3.08233	62.91	63.05	4.928	4.948
-3.2	-5314.3	99 932.16	-3.49668	67.84	67.97	5.178	5.203
-3.6	-6037.6	99 926.74	-3.91206	73.26	73.32	5.446	5.475

TABLE 7-39.- $\Delta\varphi_8$ AND ΔE_{19} FOR $N_o = 0.000395$, $H_S = 5446$ METERS, $\rho_M = 140$ 000 METERS

E_M , deg	H , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$\Delta\varphi_8$, meters	$E_M - E$, mrad	ΔE_{19} , mrad
2.0	5892.0	139 964.86	1.78467	35.14	34.77	3.758	3.725
1.6	4888.0	139 962.05	1.37324	37.95	37.73	3.958	3.930
1.2	3881.4	139 958.91	.96083	41.09	40.92	4.174	4.149
.8	2871.8	139 955.37	.54732	44.63	44.45	4.410	4.387
.4	1859.0	139 951.38	.13259	48.62	48.40	4.667	4.646
0	842.8	139 946.87	-.28349	53.13	52.88	4.948	4.930
-.4	-177.1	139 941.77	-.70109	58.23	58.00	5.255	5.244
-.8	-1201.2	139 935.98	-1.12037	64.02	63.85	5.592	5.590
-1.2	-22229.8	139 929.39	-1.54156	70.61	70.56	5.961	5.973
-1.6	-3263.5	139 921.87	-1.96487	78.13	78.24	6.368	6.397
-2.0	-4302.8	139 913.27	-2.39056	86.73	87.00	6.817	6.866
-2.4	-5348.3	139 903.39	-2.81894	96.61	96.98	7.312	7.385

TABLE 7-40.- ERRATIC BEHAVIOR OF $\Delta\rho_3$ AND ΔE_4 FOR
 $N_O = 0.000325$, $H_S = 6735$ METERS, $\rho = 40\ 000$ METERS

E, deg	H, meters	$\Delta\rho_8$, meters	$\Delta\rho_3$, meters	ΔE_{19} , mrad	ΔE_4 , mrad
1	823.5	12.27	12.11	0.928	1.068
.9	753.7	12.33	12.19	.932	1.069
.8	683.9	12.40	12.26	.935	1.067
.7	614.1	12.46	12.34	.938	1.063
.6	544.3	12.52	12.42	.941	1.056
.5	474.5	12.59	12.50	.945	1.044
.4	404.7	12.65	12.58	.948	1.027
.3	334.9	12.72	12.66	.951	1.000
.2	265.0	12.78	12.75	.954	.961
.1	195.2	12.85	12.82	.958	.900
0	125.4	12.91	12.90	.961	.800
-.1	55.6	12.98	12.97	.964	.618
-.2	-14.2	13.05	13.00	.968	.198
-.3	-84.0	13.12	12.85	.971	-1.702
-.4	-153.8	13.18	^a 15.00	.974	^a 14.831
-.5	-223.6	13.25	22.78	.978	75.358

^aSquare root of negative number evaluated for positive number.

TABLE 7-41.- EMPIRICAL COEFFICIENTS FOR $N_0 = 0.000255$, $H_S = 7892$ METERS

Altitude, meters	A_p	B_p	C_p	Percent error	A_E	B_E	C_E	Percent error
10	0.0001	200	0.5000	0.001	-0.0874	200	0.7052	1.069
20	.0002	200	.5000	.002	-.1419	200	.6951	1.352
50	.0004	133.06	.4999	.005	-.2502	200	.6738	1.317
1.E2	.0008	87.20	.4998	.012	-.3842	200	.6448	.873
2.E2	.0021	74.48	.4998	.022	-.5508	182.98	.6059	.541
5.E2	.0057	48.04	.4997	.053	-.5807	121.92	.5959	.559
1.E3	.0165	45.82	.5022	.074	-.5248	83.73	.6046	.619
2.E3	.0383	37.58	.5077	.118	-.4884	60.06	.6031	.590
5.E3	.1004	28.14	.5259	.207	-.3930	41.43	.5985	.457
1.E4	.2173	25.47	.5725	.218	-.2839	36.32	.5871	.313
2.E4	.2469	18.96	.5725	.264	-.0934	45.29	.5871	.288
5.E4	.2665	16.51	.5725	.299	.1346	14.32	.5938	.150
1.E5	.2699	16.42	.5719	.306	.1934	14.72	.5800	.156
2.E5	.2729	16.48	.5720	.305	.2426	15.96	.5800	.206
5.E5	.2754	16.54	.5720	.304	.2793	16.74	.5800	.260
1.E6	.2770	16.60	.5722	.302	.2959	17.07	.5800	.284
2.E6	.2779	16.63	.5722	.301	.3071	17.28	.5800	.299
5.E6	.2787	16.65	.5722	.300	.3167	17.46	.5800	.312
1.E7	.2793	16.68	.5723	.299	.3215	17.55	.5800	.318
2.E7	.2796	16.69	.5723	.298	.3247	17.61	.5800	.323
5.E7	.2803	16.73	.5726	.294	.3271	17.65	.5800	.326

TABLE 7-41.- Concluded.

Altitude, meters	A_p	B_p	C_p	Percent error	A_E	B_E	C_E	Percent error
1.E8	0.2803	16.77	0.5726	0.290	0.3281	17.67	0.5800	0.327

TABLE 7-42.- EMPIRICAL COEFFICIENTS FOR $N_o = 0.000325$, $H_s = 6735$ METERS

Altitude, meters	A_p	B_p	C_p	Percent error		A_E	B_E	C_E	Percent error
				—	—				
10	0.0003	200	0.5001	0.006	-0.0955	200	0.6900	1.182	
20	.0004	200	.5001	.005	-.1552	200	.6806	1.533	
50	.0007	168.41	.5000	.006	-.2716	200	.6606	1.554	
1.E2	.0012	101.22	.4999	.015	-.4079	200	.6339	1.074	
2.E2	.0027	74.87	.4999	.029	-.6017	189.40	.5909	.615	
5.E2	.0082	53.48	.5004	.065	-.6094	124.18	.5857	.628	
1.E3	.0225	49.00	.5040	.091	-.5824	88.71	.5857	.683	
2.E3	.0538	41.19	.5132	.129	-.5151	62.58	.5894	.658	
5.E3	.1239	37.75	.5732	.243	-.3929	43.71	.5857	.491	
1.E4	.2326	25.63	.5736	.239	-.2269	37.60	.5853	.329	
2.E4	.2623	19.72	.5736	.298	-.1634	58.46	.5322	.311	
5.E4	.2807	17.95	.5736	.321	.1481	14.31	.5768	.191	
1.E5	.2852	17.93	.5731	.328	.2274	16.75	.5768	.209	
2.E5	.2881	17.96	.5727	.332	.2718	17.69	.5768	.269	
5.E5	.2913	18.03	.5727	.331	.3053	18.28	.5768	.312	
1.E6	.2934	18.11	.5730	.327	.3204	18.52	.5767	.331	
2.E6	.2951	18.18	.5733	.324	.3308	18.67	.5767	.345	
5.E6	.2965	18.23	.5735	.321	.3398	18.80	.5767	.358	
1.E7	.2972	18.26	.5736	.320	.3423	18.79	.5756	.364	
2.E7	.2981	18.30	.5739	.316	.3453	18.83	.5756	.370	
5.E7	.2984	18.31	.5739	.314	.3476	18.86	.5756	.375	

TABLE 7-42.- Concluded.

Altitude, meters	A_p	B_p	C_p	Percent error	A_E	B_E	C_E	Percent error
1.E8	0.2987	18.37	0.5741	0.306	0.3486	18.86	0.5756	0.377

TABLE 7-43.- EMPIRICAL COEFFICIENTS FOR $N_0 = 0.000395$, $H_S = 5446$ METERS

Altitude, meters	A_p	B_p	C_p	Percent error	A_E	B_E	C_E	Percent error
10	0.0003	200	0.5001	0.007	-0.1096	200	0.6632	1.398
20	.0005	200	.5001	.006	-.1801	200	.6550	1.889
50	.0010	171.45	.5001	.009	-.3118	200	.6376	2.048
1.E2	.0022	125.11	.5002	.020	-.1563	200	.6140	1.517
2.E2	.0042	81.71	.5003	.043	-.6900	200	.5665	.760
5.E2	.0145	65.65	.5025	.082	-.6954	132.09	.5604	.755
1.E3	.0320	50.81	.5066	.137	-.6687	96.08	.5577	.802
2.E3	.0720	42.03	.5182	.191	-.5963	70.02	.5577	.787
5.E3	.1655	31.38	.5468	.270	-.4178	49.90	.5577	.565
1.E4	.2551	26.12	.5749	.269	-.2160	46.78	.5576	.343
2.E4	.2844	21.19	.5749	.317	.0435	8.73	.5793	.511
5.E4	.3004	20.13	.5742	.334	.1964	17.26	.5687	.288
1.E5	.3069	20.18	.5742	.337	.2634	18.83	.5687	.319
2.E5	.3090	20.16	.5731	.348	.3050	19.64	.5704	.353
5.E5	.3126	20.24	.5730	.350	.3534	21.11	.5813	.487
1.E6	.3146	20.28	.5730	.351	.3664	21.10	.5813	.452
2.E6	.3158	20.35	.5730	.352	.3752	21.09	.5813	.440
5.E6	.3169	20.41	.5730	.354	.3832	21.02	.5813	.443
1.E7	.3176	20.42	.5730	.354	.3869	21.02	.5813	.452
2.E7	.3180	20.44	.5730	.354	.3897	20.98	.5813	.460
5.E7	.3182	20.47	.5730	.352	.3917	20.97	.5813	.469

TABLE 7-43.- Concluded.

Altitude, meters	A_p	B_p	C_p	Percent error	A_E	B_E	C_E	Percent error
1.88	0.3183	20.48	0.5730	0.343	0.3925	20.96	0.5813	0.473

TABLE 7-44.- VALUES OF $\Delta\rho_{10}$ FOR $N_o = 0.000255$, $H_S = 7892$ METERS

$H =$	1.E5 meters			1.E6 meters			1.E7 meters			1.E8 meters		
	$\rho_M - \rho$, meters	$\Delta\rho_{10}$, meters										
0	83.0	82.5	84.0	83.8	84.4	84.3	84.3	84.3	84.5	84.4	84.5	84.4
.1	79.1	78.7	80.0	79.9	80.3	80.3	80.3	80.3	80.4	80.4	80.4	80.4
.2	75.5	75.3	76.3	76.3	76.6	76.6	76.6	76.6	76.7	76.7	76.7	76.7
.3	72.1	72.1	72.9	73.0	73.1	73.3	73.1	73.3	73.2	73.3	73.2	73.3
.4	69.1	69.1	69.7	69.9	69.9	70.1	69.9	70.1	70.0	70.0	70.0	70.2
.5	66.2	66.3	66.8	67.0	67.0	67.2	67.0	67.2	67.0	67.0	67.0	67.2
.7	61.0	61.2	61.5	61.7	61.7	61.9	61.7	61.9	61.7	61.7	61.7	61.9
1	54.5	54.7	54.9	55.1	55.0	55.2	55.0	55.2	55.0	55.0	55.0	55.2
2	39.6	39.6	39.8	39.7	39.8	39.7	39.8	39.7	39.8	39.8	39.8	39.7
3	30.7	30.6	30.7	30.6	30.8	30.6	30.8	30.6	30.8	30.8	30.8	30.6
4	24.8	24.7	24.9	24.8	24.9	24.8	24.9	24.8	24.9	24.9	24.9	24.8
5	20.8	20.7	20.8	20.7	20.8	20.7	20.8	20.7	20.8	20.8	20.8	20.7
7	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6
10	11.2	11.3	11.2	11.3	11.2	11.3	11.2	11.3	11.2	11.2	11.2	11.3

TABLE 7-45.- VALUES OF $\Delta\rho_{10}$ FOR $N_o = 0.000325$, $H_S = 6735$ METERS

$H =$	1.E5 meters			1.E6 meters			1.E7 meters			1.E8 meters		
	$\rho_M - \rho$, meters	$\Delta\rho_{10}$, meters										
0	106.7	106.0	108.8	108.5	109.6	109.2	109.6	109.2	109.8	109.8	109.3	109.3
0.1	100.6	100.1	102.4	102.3	103.1	102.8	103.1	102.8	103.3	103.3	102.9	102.9
.2	95.1	94.8	96.7	96.6	97.2	97.1	97.2	97.1	97.4	97.4	97.2	97.2
.3	90.1	89.9	91.5	91.5	91.9	91.9	91.9	91.9	92.1	92.1	92.0	92.0
.4	85.6	85.5	86.8	86.9	87.2	87.2	87.2	87.2	87.3	87.3	87.3	87.3
.5	81.4	81.5	82.4	82.7	82.8	82.8	82.8	82.9	82.9	82.9	83.0	83.0
.7	74.1	74.3	74.9	75.2	75.2	75.4	75.2	75.4	75.2	75.2	75.4	75.4
1	65.1	65.3	65.7	65.9	65.8	66.0	65.8	66.0	65.9	65.9	66.1	66.1
2	45.7	45.6	45.9	45.8	45.9	45.8	45.9	45.8	45.9	45.9	45.8	45.8
3	34.7	34.5	34.8	34.6	34.8	34.6	34.8	34.6	34.8	34.8	34.6	34.6
4	27.8	27.7	27.8	27.7	27.8	27.7	27.8	27.7	27.8	27.8	27.7	27.7
5	23.1	23.0	23.1	23.0	23.1	23.0	23.1	23.0	23.1	23.1	23.0	23.0
7	17.1	17.2	17.1	17.2	17.1	17.2	17.1	17.2	17.1	17.1	17.2	17.2
10	12.3	12.4	12.3	12.4	12.3	12.4	12.3	12.4	12.3	12.3	12.4	12.4

TABLE 7-46.- VALUES OF $\Delta\rho_{10}$ FOR $N_O = 0.000395$, $H_S = 5446$ METERS

$H =$	1.E5 meters			1.E6 meters			1.E7 meters			1.E8 meters		
	$\rho_M - \rho$, meters	$\Delta\rho_{10}$, meters										
0	137.7	136.8	142.1	142.6	143.6	144.1	144.1	144.1	144.1	144.1	144.1	144.4
.1	126.9	126.0	130.4	130.5	131.7	131.8	132.1	132.1	132.1	132.1	132.0	132.0
.2	117.5	116.9	120.5	120.5	121.5	121.5	121.8	121.8	121.8	121.8	121.7	121.7
.3	109.4	109.0	111.9	112.0	112.7	112.8	113.0	113.0	113.0	113.0	113.0	113.0
.4	102.3	102.0	104.3	104.6	105.1	105.3	105.3	105.3	105.3	105.3	105.3	105.5
.5	95.9	95.9	97.7	98.1	98.3	98.7	98.5	98.5	98.5	98.5	98.8	98.8
.7	85.3	85.4	86.6	87.1	87.0	87.5	87.1	87.1	87.1	87.1	87.7	87.7
1	72.9	73.1	73.7	74.2	74.0	74.5	74.1	74.1	74.1	74.1	74.6	74.6
2	48.4	48.3	48.6	48.6	48.7	48.7	48.7	48.7	48.7	48.7	48.8	48.8
3	35.8	35.6	35.9	35.7	35.9	35.8	35.9	35.9	35.9	35.9	35.8	35.8
4	28.2	28.1	28.3	28.2	28.3	28.2	28.3	28.3	28.3	28.3	28.3	28.3
5	23.2	23.2	23.2	23.2	23.2	23.2	23.3	23.3	23.3	23.3	23.3	23.3
7	17.1	17.1	17.1	17.2	17.1	17.2	17.1	17.2	17.1	17.1	17.2	17.2
10	12.2	12.3	12.2	12.3	12.2	12.3	12.2	12.3	12.2	12.2	12.3	12.3

TABLE 7-47.- VALUES OF ΔE_{21} FOR $N_o = 0.000255$, $H_S = 7892$ METERS

$H =$	1.E5 meters			1.E6 meters			1.E7 meters			1.E8 meters		
	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad
0	8.34	8.32	9.47	9.44	9.85	9.83	9.96	9.88				
0.1	8.00	7.98	9.06	9.04	9.43	9.40	9.53	9.45				
.2	7.68	7.66	8.69	8.67	9.03	9.01	9.12	9.06				
.3	7.39	7.37	8.34	8.32	8.66	8.64	8.75	8.69				
.4	7.11	7.10	8.01	8.00	8.31	8.30	8.40	8.35				
.5	6.85	6.84	7.71	7.70	7.99	7.98	8.07	8.03				
.7	6.37	6.37	7.15	7.15	7.40	7.40	7.47	7.44				
1	5.76	5.76	6.44	6.44	6.65	6.65	6.71	6.68				
2	4.30	4.30	4.76	4.75	4.89	4.88	4.93	4.90				
3	3.39	3.38	3.72	3.71	3.81	3.80	3.83	3.81				
4	2.77	2.76	3.03	3.03	3.10	3.09	3.11	3.10				
5	2.34	2.33	2.55	2.55	2.59	2.59	2.60	2.60				
7	1.76	1.76	1.91	1.92	1.94	1.95	1.95	1.96				
10	1.27	1.28	1.38	1.39	1.41	1.40	1.41	1.41				

TABLE 7-48.- VALUES OF ΔE_{21} FOR $N_O = 0.000325$, $H_S = 6735$ METERS

$H =$	1.E5 meters			1.E6 meters			1.E7 meters			1.E8 meters		
	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad
0	12.28	12.33	13.82	13.84	14.36	14.42	14.52	14.50				
0.1	11.70	11.72	13.14	13.14	13.64	13.69	13.78	13.77				
.2	11.16	11.16	12.51	12.51	12.97	13.02	13.10	13.10				
.3	10.66	10.66	11.93	11.93	12.36	12.41	12.48	12.48				
.4	10.20	10.20	11.39	11.40	11.80	11.85	11.91	11.92				
.5	9.77	9.78	10.90	10.92	11.28	11.33	11.38	11.39				
.7	9.01	9.02	10.02	10.04	10.35	10.39	10.44	10.45				
1	8.04	8.06	8.91	8.93	9.18	9.21	9.25	9.26				
2	5.85	5.86	6.41	6.40	6.56	6.56	6.61	6.58				
3	4.54	4.54	4.93	4.92	5.03	5.02	5.06	5.04				
4	3.68	3.68	3.98	3.98	4.05	4.04	4.07	4.06				
5	3.08	3.08	3.32	3.33	3.37	3.38	3.38	3.39				
7	2.30	2.31	2.47	2.49	2.50	2.52	2.51	2.53				
10	1.66	1.67	1.77	1.79	1.79	1.81	1.79	1.81				

TABLE 7-49.- VALUES OF ΔE_{21} FOR $N_0 = 0.000395$, $H_S = 5446$ METERS

$H =$	1.E5 meters			1.E6 meters			1.E7 meters			1.E8 meters		
	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad	$E_M - E$, mrad	ΔE_{21} , mrad
0	18.50	18.84	20.65	20.69	21.43	21.50	21.66	21.56	21.66	21.50	21.56	21.56
0.1	17.37	17.52	19.34	19.30	20.05	20.08	20.25	20.17	20.25	20.08	20.17	20.17
.2	16.37	16.41	18.18	18.11	18.82	18.85	19.00	18.95	19.00	18.82	18.95	18.95
.3	15.46	15.47	17.14	17.07	17.72	17.76	17.89	17.87	17.89	17.72	17.87	17.87
.4	14.65	14.64	16.20	16.15	16.74	16.78	16.89	16.89	16.89	16.74	16.89	16.89
.5	13.91	13.90	15.35	15.31	15.84	15.90	15.98	15.98	15.98	15.84	15.98	16.00
.7	12.62	12.62	13.88	13.86	14.30	14.35	14.41	14.41	14.41	13.86	14.41	14.44
1	11.05	11.08	12.10	12.08	12.43	12.46	12.52	12.53	12.52	12.10	12.52	12.53
2	7.71	7.73	8.34	8.30	8.52	8.47	8.57	8.51	8.57	8.30	8.57	8.51
3	5.85	5.86	6.28	6.24	6.39	6.34	6.42	6.36	6.42	6.24	6.34	6.36
4	4.68	4.70	5.00	4.99	5.07	5.05	5.09	5.07	5.09	4.99	5.05	5.07
5	3.89	3.91	4.13	4.14	4.19	4.19	4.20	4.20	4.20	4.13	4.19	4.20
7	2.88	2.91	3.05	3.07	3.08	3.11	3.09	3.11	3.09	3.05	3.11	3.11
10	2.06	2.08	2.17	2.19	2.21	2.21	2.19	2.22	2.19	2.08	2.21	2.22

8.0 SENSITIVITY TO N_o

Errors in N_o cause errors in the estimate of the refraction corrections. For processing radar tracking data at Johnson Space Center, mean monthly values of N_o are used. A study of reference 5 reveals that N_o is changed from month to month by an average amount of 1.7 percent with a maximum of 7.5 percent observed at one station. This is not to imply that using mean monthly values is incorrect. In fact, they may represent the total atmosphere above the station better than an estimate made at the station near the track time. Note that diurnal variations in N_o at inland stations may be as much as 6 percent from the mean daily value. A strong weather front passage can change N_o by 14 percent. Thus, one can expect a 10% error in the mean monthly N_o value of approximately 2.5 percent. That is,

$$\sigma_{N_o} = 0.025 N_o \quad (8.1)$$

Equation (7.19) gave the range refraction correction for large elevation angles ($E > 5$ degrees for error analyses)

$$\Delta\rho_4 = \frac{N_o H^*}{\sin E} \quad (8.2)$$

where

$$H^* = (1 - \text{EXP}(-H/H_S))H_S$$

$H^* = H_S$ for $H > 10^5$ meters = 54 n. mi. $H^* \approx H$ for H small. For H small,

$$\Delta\rho_4 = \frac{N_o H}{\sin E}$$

and

$$\frac{\partial\Delta\rho_4}{\partial N_o} = \frac{H}{\sin E} = \frac{1}{N_o} \Delta\rho_4$$

or

$$\delta (\Delta\rho_4) = \frac{\delta N_O}{N_O} \Delta\rho_4$$

In general,

$$\delta(\Delta\rho) = K_p \frac{\delta N_O}{N_O} \Delta\rho$$

(8.3)

where

$$K_p = \frac{\partial \Delta\rho}{\partial N_O} \frac{N_O}{\Delta\rho}$$

(8.4)

Note that $K_p = 1$ for H small and E large. For H large, $H^* = H_S$, and for $E > 5$ degrees.

$$\Delta\rho_4 = \frac{N_O H_S}{\sin E}$$

$$\frac{\partial \Delta\rho_4}{\partial N_O} = \frac{H_S}{\sin E} + \frac{N_O}{\sin E} \frac{\partial H_S}{\partial N_O}$$

Thus, in this case

$$K_p = 1 + \frac{N_O}{H_S} \frac{\partial H_S}{\partial N_O} \quad (8.5)$$

where, from equation (2.6),

$$\frac{\partial H_S}{\partial N_O} = 35.05462 \cdot 10^6 - 0.3007245 \cdot 10^{12} N_O + 4.215282 \cdot 10^{14} N_O^2 \quad (8.6)$$

For $N_O = 0.000255$, $H_S = 7892$ meters

$$K_P = 0.541$$

For $N_O = 0.000325$, $H_S = 6735$ meters

$$K_P = 0.124$$

For $N_O = 0.000395$, $H_S = 5446$ meters

$$K_P = -0.303$$

For E large, equation (7.31) was

$$\Delta E_{10} = \frac{N_O \cos E}{\sin E} \left(1 - \frac{H^*/\rho}{\sin E} \right) \quad (8.7)$$

For orbital altitudes, the $H^* = H_S$ term is small compared to 1, so approximately

$$\frac{\partial \Delta E}{\partial N_O} = \frac{\cos E}{\sin E} \left(1 - \frac{H_S/\rho}{\sin E} \right)$$

or

$$\frac{\partial \Delta E}{\partial N_O} = \frac{1}{N_O} \Delta E \quad (8.8)$$

or

$$\delta(\Delta E) = \frac{\delta N_o}{N_o} \Delta E \quad (8.9)$$

In general,

$$\delta(\Delta E) = K_E \frac{\delta N_o}{N_o} \Delta E \quad (8.10)$$

where

$$K_E = \frac{\partial \Delta E}{\partial N_o} \frac{N_o}{\Delta E} \quad (8.11)$$

Note that $K_E = 1$ for E large at orbital altitudes.

Tables 8-1 and 8-2 show experimentally derived values of $\partial \Delta \rho / \partial N_o$ and $\partial \Delta E / \partial N_o$. Tables 8-3 and 8-4 show experimentally derived values of K_ρ and K_E . For $H = 1.E6$ meters, the numerical (secant) partials were obtained from $\Delta \rho_{10}$ and ΔE_{21} . For the other values of H , the partials were obtained by using $\Delta \rho_3$ and $\Delta \rho_4$.

To get some approximate statistical equations, the RMS values of K_ρ and K_E were obtained to give

$$\sigma_{\Delta \rho} = 0.66 (\sigma_{N_o}/N_o) \Delta \rho \quad (8.12)$$

$$\sigma_{\Delta E} = 1.55 (\sigma_{N_o}/N_o) \Delta E \quad (8.13)$$

where, from equation (8.1)

$$\sigma_{N_0}/N_0 = 0.025$$

The maximum value of $\Delta\rho$ is about 144 meters. The maximum value of ΔE is approximately 22 milliradians. Thus

$$\sigma_{\Delta\rho} < 2.4 \text{ meters}$$

$$\sigma_{\Delta E} < 0.85 \text{ mrad}$$

Typical C-band radar biases are 10 meters and 0.1 milliradians. Thus, the elevation angle error can be significant at low elevation angles. For $E > 5$ degrees, $\Delta\rho \leq 23$ meters and $\Delta E \leq 4.2$ milliradians. Thus for $E > 5$ degrees.

$$\sigma_{\Delta\rho} < 0.4 \text{ meters}$$

$$\sigma_{\Delta E} < 0.16 \text{ mrad}$$

TABLE 8-1.- $\partial\Delta p/\partial N_O$ METERS

H, meters	N_O	$E_M = 1^\circ$	$E_M = 5^\circ$	$E_M = 10^\circ$
1.E6	0 .000255	151 540	48 600	25 190
1.E6	.000325	76 340	14 070	5 820
1.E6	.000395	30 350	-11 740	-8 760
1.E4	.000255	126 760	47 790	25 130
1.E4	.000325	90 220	30 790	15 230
1.E4	.000395	55 790	10 360	3 920
1.E3	.000255	43 420	10 350	5 250
1.E3	.000325	41 710	9 890	5 010
1.E3	.000395	39 470	14 780	7 440

TABLE 8-2.- $\partial\Delta E/\partial N_O$ MILLIRADIANS

H, meters	N_O	$E_M = 1^\circ$	$E_M = 5^\circ$	$E_M = 10^\circ$
1.E6	0 .000255	29 120	10 592	5 590
1.E6	.000325	32 746	11 038	5 703
1.E6	.000395	40 730	11 450	5 775
1.E4	.000255	16 296	5 930	3 121
1.E4	.000325	22 406	7 886	4 126
1.E4	.000395	31 562	10 304	5 283
1.E3	.000255	5 233	1 023	491
1.E3	.000325	7 647	1 484	732
1.E3	.000395	11 051	3 936	1 994

TABLE 8-3.- VALUES OF K_p

H, meters	N_o	$E_M = 1^\circ$	$E_M = 5^\circ$	$E_M = 10^\circ$
1.E6	0.000255	0.702	0.598	0.569
1.E6	.000325	.376	.198	.152
1.E6	.000395	.162	-.199	-.282
1.E4	.000255	.722	.786	.779
1.E4	.000325	.541	.545	.512
1.E4	.000395	.353	.205	.149
1.E3	.000255	.954	.972	.972
1.E3	.000325	.916	.937	.937
1.E3	.000395	.863	.782	.774

TABLE 8-4.- VALUES OF K_E

H, meters	N_o	$E_M = 1^\circ$	$E_M = 5^\circ$	$E_M = 10^\circ$
1.E6	0.000255	1.153	1.060	1.027
1.E6	.000325	1.192	1.079	1.037
1.E6	.000395	1.331	1.092	1.040
1.E4	.000255	1.422	1.299	1.301
1.E4	.000325	1.673	1.552	1.550
1.E4	.000395	1.943	1.771	1.745
1.E3	.000255	1.629	1.411	1.427
1.E3	.000325	1.943	1.775	1.831
1.E3	.000395	2.226	2.103	2.169

9.0 SENSITIVITY TO R_o

The radius of curvature of the Earth varies widely with latitude and direction. The north-south radius of curvature of the Earth is given by

$$R_C = \frac{(1 - e)^2 R_E}{(1 - (1 - (1 - e)^2) \sin^2 \phi)^{3/2}} \quad (9.1)$$

where

ϕ = geodetic latitude

e = ellipticity or flattening = 1/298.3

R_E = 6 378 166 meters, equatorial radius

R_p = 6 356 784 meters, polar radius = $(1 - e) R_E$

At $\phi = 0$ degree, the equator, $R_C = 6 335 474$ meters, and the East-West curvature is 6 378 166 meters, a difference of approximately 40 kilometers. As ϕ increases, so does R_C . At $\phi = 90$ degrees, the North Pole, $R_C = 6 399 620$ meters.

Previously, it was recommended that

$$R_o = 6 378 165 \text{ meters} = 1 \text{ Earth radius}$$

The typical radius of curvature is about $R_o \pm 13$ kilometers σ . The maximum sensitivity to R_o occurs at $H = 10^8$ meters, at $E_{Mi} = 0$ degree, and for $N_o = 0.000395$. In this case,

$$\frac{\partial \Delta E}{\partial R_o} = 0.0028 \frac{\text{mrad}}{\text{km}} \quad (9.2)$$

$$\frac{\partial \Delta \rho}{\partial R_o} = 0.026 \frac{\text{meter}}{\text{km}} \quad (9.3)$$

For $\delta R_o = 13$ kilometers, this amounts to a 0.04 milliradian error in ΔE and a 0.3-meter error in $\Delta \rho$. The error is negligible for $E_{Mi} > 2$ degrees. For $H = 10^4$ meters, the maximum errors are 0.02 milliradians and 0.2 meter. For

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$H = 10^4$ meters and $E_{Mi} > 1$ degree, the errors are negligible. For $H = 10^3$ meters, the errors are always negligible.

10.0 QUESTIONS CONCERNING ATMOSPHERIC SCALE HEIGHT

For an exponential atmosphere, the modulus of refraction is given by

$$N = N_0 \exp(-h/H_S) \quad (10.1)$$

where $N_0 = N$ at $h = 0$. The h is altitude above R_0 . Given a value of N at some arbitrary altitude, the value of the atmospheric scale height H_S , can be computed.

Goddard Space Flight Center, which supplies values of H_S to Johnson Space Center, uses the value of N at $h = 1000$ meters above the tracking site to determine H_S . This value of N is determined empirically from the equations given in reference 1, which was written in 1959 before satellites were being tracked in orbit. The equation is

$$N_{1000} = N_0 - 7.32 \cdot 10^{-6} \exp(5577 N_0) \quad (10.2)$$

Atmospheric scale height is then computed from

$$H_{S1} = 1000/\ln(N_0/N_{1000}) \text{ meters} \quad (10.3)$$

At Patrick Air Force Base the yearly mean value of N at 100 000 feet ($\bar{N}_{100\ 000}$) is used to compute H_S for use at the Eastern Test Range.

$$H_{S2} = 100\ 000/\ln(N_0/\bar{N}_{100\ 000}) \text{ feet} \quad (10.4)$$

where

$$\bar{N}_{100\ 000} = 3.36 \cdot 10^{-6} \quad (10.5)$$

$\bar{N}_{100\ 000}$ was obtained only for Cape Canaveral.

Using a summertime Cape value of $N_0 = 0.000395$ gives $H_{S1} = 5446$ meters and $H_{S2} = 6397$ meters (a substantial difference). As will be discussed later in this document, H_{S2} gives the more accurate refraction corrections when compared with those obtained using an actual Cape atmosphere. A comparison of the refraction corrections obtained with H_{S1} and H_{S2} is seen in tables 10-1, 10-2, and 10-3. The tables show that there are significant differences in the refraction corrections, particularly for the elevation angle correction. For a typical C-band radar, the hardware 10° biases are about 10 meters for range and 0.1 milliradian for elevation angle. For $H = 1.E6$ meter and $E_M = 3$ degrees,

the ΔE difference, $\delta(\Delta E)$, is 0.18 milliradian, nearly twice the hardware bias. This causes an altitude error of approximately 620 m = 2000 feet. At $H = 1.E4$ meter and $E_{Mi} = 3$ degrees, $\delta(\Delta E) = 0.34$ milliradian, over 3 times greater than the hardware bias. Even at $H = 1.E4$ meter and $E_{Mi} = 10$ degrees, $\delta(\Delta E) = 0.11$ milliradian. At a range 60 000 meters and $E_{Mi} = 0$ degree, $\delta(\Delta E) = 0.32$ giving an altitude error of 19 meters = 62 feet, which could occur during the landing of a Space Shuttle at Kennedy Space Center. At $E_{Mi} = 1$ degree, table 10-1 and 10-2 differences are $\delta(\Delta E) = 0.65$ and 0.74 milliradians.

Table 10-4 shows the yearly mean values of N at altitudes up to 110 000 feet for Cape Canaveral. This average profile was computed from 4 years (1963 through 1966) of accumulated radiosonde data furnished by Pan Am Meteorological Data Reduction. Tables 10-5 and 10-6 show mean monthly values of N for August and December at the Cape. Figure 10-1 shows a plot of $\log(N \cdot 10^6)$ versus altitude for the mean yearly profile. This plot would show a straight line if the atmosphere were truly exponential (if eq. (10.1) were exact). As shown in figure 10-1, the plot is not exactly a straight line.

Refraction corrections based on this yearly mean atmosphere can be computed using the integral equations for Δp_7 and ΔE_{18} given in section 5.0. The integral equations may be accurately solved by using five-point Gaussian quadrature. Figure 10-1 shows the quadrature points at which the integrands are evaluated for orbital refraction corrections, $H = 1.E6$ meters = 540 n. mi. Note that the heavily weighted center quadrature point occurs at $h = 15\ 000$ feet, and the highest point is at 69 500 feet. This is important because values of N above 69 500 feet are not very significant, and the most significant value of N occurs at 15 000 feet = 4600 meters (not at 100 000 feet and not at 1000 meters). This clearly indicates a third method of computing H_S .

$$H_{S3} = 4600/\ln(N_0/\bar{N}_{4600}) \text{ meters} \quad (10.6)$$

or

$$H_{S3} = 15\ 000/\ln(N_0/N_{15\ 000}) \text{ feet} \quad (10.7)$$

where \bar{N}_{4600} is the mean monthly value of N at $h = 4600$ meters above the station, or $N_{15\ 000}$ is the mean monthly value of N at $h = 15\ 000$ feet above the tracking site.

To help evaluate which method of computing H_S is best, Δp_7 and ΔE_{18} were computed using the values shown in table 10-4, the mean yearly values of N at Cape Canaveral. Table 10-7 then compares these values of Δp_7 and ΔE_{18} with those obtained using the three methods of computing H_S . Table 10-7 shows that H_{S3} is clearly the best, H_{S2} is average, and H_{S1} is poor. However, in atmospheres that deviate even more from the exponential atmosphere, H_{S2} may not provide as good results since H_{S2} is evaluated with N at $h = 100\ 000$ feet, well outside the quadrature range. H_{S2} does have one advantage over H_{S3} .

Tables 10-4, 10-5, and 10-6 show that the value of N at 100 000 feet is relatively independent of the month. And this constant value may apply to other stations at similar latitudes. If this is true, then high altitude meteorological data need not be obtained for every tracking site. It is clear that these matters need further study.

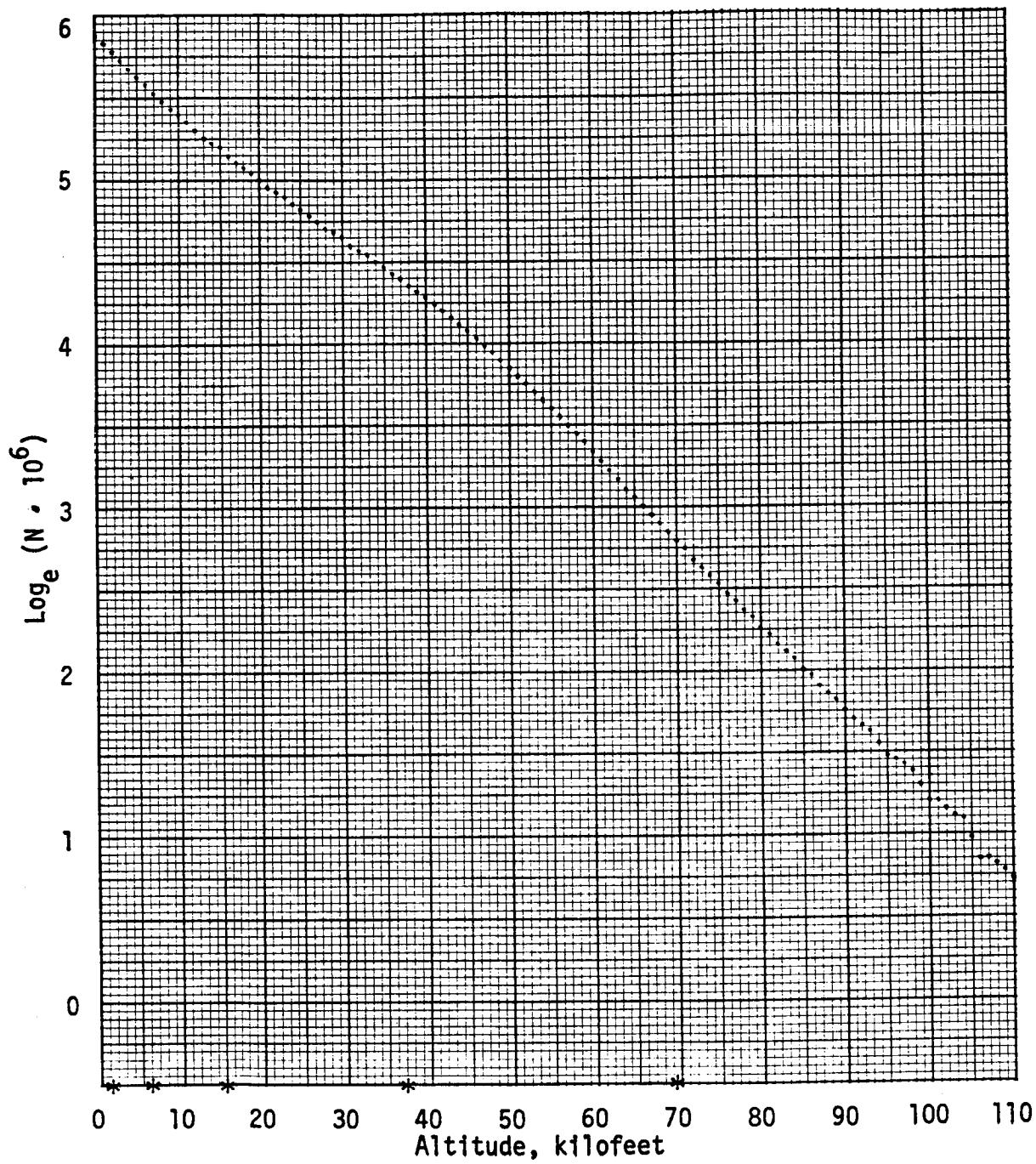


Figure 10-1.- $\text{Log}_e(N \cdot 10^6)$ versus altitude
for yearly mean values

*Gaussian quadrature points

TABLE 10-1.- REFRACTION CORRECTIONS FOR $H = 1.1E6$ METERS (540 NAUTICAL MILES),
 $N_o = 0.000395$, $H_S = 5446$ METERS, AND $H_S = 6397$ METERS

E_{M1} , deg	ρ , meters	E , deg	$H_S = 5446$ meters		$H_S = 6397$ meters	
			$\Delta\phi_7$, meters	$\Delta\phi_{18}$, mrad	$\Delta\phi_7$, meters	$\Delta\phi_{18}$, mrad
0.1	3 822 890.6	-1.0083	130.2	19.33	139.3	17.93
.2	3 803 825.3	-.8417	120.1	18.18	129.1	16.89
.3	3 785 646.4	-.6820	111.5	17.14	120.3	15.97
.4	3 768 225.1	-.5283	103.9	16.20	112.6	15.13
.5	3 751 457.8	-.3797	97.3	15.36	105.9	14.38
.7	3 719 562.7	-.0952	86.1	13.88	94.4	13.06
1	3 674 936.2	.3070	73.3	12.10	81.1	11.45
2	3 543 398.8	1.5221	48.2	8.35	54.5	8.03
3	3 426 768.0	2.6402	35.5	6.29	40.6	6.11
4	3 318 776.8	3.7136	27.9	5.00	32.2	4.90
5	3 216 948.1	4.7632	22.9	4.14	26.6	4.07
7	3 027 621.9	6.8253	16.8	3.05	19.6	3.02
10	2 772 989.0	9.8756	12.0	2.17	14.0	2.16
20	2 124 626.1	19.9386	6.1	1.07	7.2	1.07
30	1 703 701.9	29.9612	4.2	.68	4.9	.68
40	1 429 322.9	39.9732	3.3	.47	3.8	.47
50	1 248 558.0	49.9811	2.8	.33	3.2	.33
70	1 054 833.1	69.9918	2.2	.14	2.6	.14
90	10 ⁶ 90	2.1	0	2.5	0	0

TABLE 10-2.- REFRACTION CORRECTIONS FOR $H = 1.14$ METERS (33 000 FEET),
 $N_o = 0.000395$, $H_S = 5446$ METERS, AND $H_S = 6397$ METERS

E _{M1} , deg	ρ , meters	$H_S = 5446$ meters				$H_S = 6397$ meters			
		E, deg		$\Delta\phi_7$, meters	ΔE_{18} , mrad	$\Delta\phi_7$, meters	ΔE_{18} , mrad	$\rho\delta(\Delta E_{18})$, meters	
0.1	429 161.91	-0.59151		112.2	12.06	118.4	10.71		580
.2	410 638.47	-.44803		104.1	11.30	110.2	10.05		510
.3	393 361.79	-.30910		97.0	10.63	103.1	9.46		460
.4	377 199.36	-.17404		90.8	10.02	96.7	8.93		410
.5	362 042.56	-.04232		85.2	9.46	91.0	8.45		370
.7	334 396.53	.21279		75.7	8.50	81.3	7.61		300
1	298 586.23	.57930		64.6	7.34	69.8	6.60		220
2	214 039.27	1.71178		42.5	4.92	46.3	4.45		100
3	162 802.03	2.79198		31.1	3.63	34.1	3.29		60
4	129 816.89	3.83683		24.3	2.85	26.7	2.59		30
5	107 326.43	4.86643		19.9	2.33	21.9	2.12		20
7	79 182.26	6.90264		14.5	1.70	16.0	1.55		10
10	56 572.62	9.93132		10.3	1.20	11.4	1.09		10
20	29 113.60	19.96640		5.3	.58	5.8	.53		0
30	19 965.99	29.97878		3.6	.37	4.0	.34		0
40	15 544.69	39.98539		2.8	.25	3.1	.23		0
50	13 048.85	49.98971		2.4	.18	2.6	.16		0
70	10 640.98	69.99553		1.9	.08	2.1	.07		0
90	10 ⁴ 90			1.8	0	2.0	0		0

TABLE 10-3.- REFRACTION CORRECTIONS FOR $\rho_M = 60\ 000$ METERS,
 $N_0 = 0.000395$, $H_S = 5446$ METERS AND $H_S = 6397$ METERS

E_{Mi} , deg	H , meters	ρ , meters	$H_S = 5446$ meters			$H_S = 6397$ meters		
			E , deg	$\Delta\rho_8$, meters	ΔE_{19} , mrad	$\Delta\rho_8$, meters	ΔE_{19} , mrad	$\rho\delta(\Delta E_{19})$, meters
6	6458.4	59 986.0	5.9129	13.8	1.51	15.0	1.36	9
5	5411.9	59 984.9	4.9080	15.1	1.60	16.1	1.42	11
4	4363.2	59 983.6	3.9027	16.4	1.70	17.3	1.50	12
3	3312.6	59 982.1	2.8969	17.9	1.80	18.6	1.57	14
2	2260.4	59 980.5	1.8905	19.5	1.91	20.1	1.66	15
1	1206.8	59 978.6	.8836	21.4	2.03	21.7	1.75	17
0	152.2	59 976.5	-.1240	23.5	2.16	23.6	1.84	19
-1	-903.2	59 974.1	-1.1323	25.9	2.31	25.6	1.95	22
-2	-1959.3	59 971.3	-2.1414	28.7	2.47	27.9	2.06	25
-3	-3015.6	59 968.2	-3.1514	31.9	2.65	30.5	2.19	28
-4	-4071.9	59 964.6	-4.1625	35.5	2.84	33.4	2.32	31
-5	-5128.1	59 960.5	-5.1746	39.6	3.05	36.3	2.46	35
-6	-6183.9	59 955.7	-6.1880	44.2	3.28	40.1	2.62	40

TABLE 10-4.- YEARLY MEAN VALUES OF $N \cdot 10^6$ AT CAPE CANAVERAL

H , ft	$N \cdot 10^6$	H , ft	$N \cdot 10^6$	H , ft	$N \cdot 10^6$
0	355.89	22 000	138.14	44 000	61.29
1 000	337.94	23 000	133.25	45 000	58.71
2 000	321.79	24 000	128.53	46 000	56.29
3 000	306.38	25 000	124.01	47 000	53.77
4 000	291.22	26 000	119.67	48 000	51.39
5 000	277.00	27 000	115.51	49 000	49.06
6 000	263.23	28 000	111.56	50 000	46.83
7 000	250.54	29 000	107.63	51 000	44.65
8 000	239.09	30 000	103.91	52 000	42.55
9 000	228.40	31 000	100.31	53 000	40.49
10 000	218.52	32 000	96.82	54 000	38.51
11 000	209.51	33 000	93.43	55 000	36.57
12 000	201.17	34 000	90.16	56 000	34.70
13 000	193.38	35 000	86.98	57 000	32.91
14 000	186.05	36 000	83.87	58 000	31.20
15 000	178.98	37 000	80.84	59 000	29.54
16 000	172.24	38 000	77.88	60 000	27.98
17 000	165.92	39 000	74.99	61 000	26.46
18 000	159.88	40 000	72.15	62 000	25.05
19 000	154.13	41 000	69.35	63 000	23.69
20 000	148.58	42 000	66.63	64 000	22.42
21 000	143.24	43 000	63.92	65 000	21.23

TABLE 10-4.- Concluded

H, ft	N • 10 ⁶	H, ft	N • 10 ⁶	H, ft	N • 10 ⁶
66 000	20.12	81 000	9.24	96 000	4.31
67 000	19.05	82 000	8.68	97 000	4.15
68 000	18.07	83 000	8.33	98 000	3.99
69 000	17.13	84 000	7.92	99 000	3.65
70 000	16.25	85 000	7.48	100 000	3.36
71 000	15.42	86 000	7.20	101 000	3.32
72 000	14.62	87 000	6.73	102 000	3.20
73 000	13.86	88 000	6.41	103 000	3.05
74 000	13.20	89 000	6.19	104 000	2.98
75 000	12.53	90 000	5.81	105 000	2.67
76 000	11.85	91 000	5.46	106 000	2.34
77 000	11.31	92 000	5.29	107 000	2.36
78 000	10.67	93 000	5.10	108 000	2.27
79 000	10.23	94 000	4.72	109 000	2.18
80 000	9.63	95 000	4.41	110 000	2.08

TABLE 10-5.- AUGUST MEAN VALUES OF $N \cdot 10^6$ AT CAPE CANAVERAL

H, ft	$N \cdot 10^6$	H, ft	$N \cdot 10^6$	H, ft	$N \cdot 10^6$
0	378.63	22 000	139.26	44 000	62.82
1 000	361.52	23 000	134.16	45 000	60.35
2 000	341.08	24 000	129.28	46 000	57.86
3 000	322.90	25 000	124.63	47 000	55.38
4 000	306.76	26 000	120.10	48 000	53.03
5 000	292.29	27 000	115.73	49 000	50.58
6 000	277.80	28 000	111.59	50 000	48.21
7 000	264.30	29 000	107.66	51 000	45.89
8 000	252.21	30 000	103.93	52 000	43.83
9 000	240.31	31 000	100.35	53 000	41.48
10 000	229.29	32 000	95.90	54 000	39.24
11 000	218.95	33 000	93.56	55 000	37.45
12 000	209.46	34 000	90.39	56 000	35.41
13 000	201.07	35 000	87.32	57 000	33.50
14 000	192.84	36 000	84.30	58 000	31.69
15 000	184.51	37 000	81.36	59 000	29.94
16 000	176.55	38 000	78.45	60 000	28.29
17 000	169.50	39 000	75.71	61 000	26.77
18 000	162.82	40 000	73.08	62 000	25.29
19 000	156.50	41 000	70.43	63 000	24.01
20 000	150.37	42 000	67.89	64 000	22.71
21 000	144.73	43 000	65.33	65 000	21.46

TABLE 10-5.- Concluded

H, ft	N • 10 ⁶	H, ft	N • 10 ⁶	H, ft	N • 10 ⁶
66 000	20.37	81 000	9.38	96 000	4.32
67 000	19.33	82 000	9.00	97 000	4.26
68 000	18.33	83 000	8.38	98 000	3.96
69 000	17.38	84 000	8.11	99 000	3.95
70 000	16.46	85 000	7.56	100 000	3.32
71 000	15.64	86 000	7.36	101 000	3.31
72 000	14.94	87 000	7.00	102 000	3.31
73 000	14.20	88 000	6.41	103 000	3.00
74 000	13.39	89 000	6.35	104 000	2.97
75 000	12.77	90 000	6.00	105 000	2.93
76 000	12.17	91 000	5.54	106 000	2.32
77 000	11.42	92 000	5.36	107 000	2.28
78 000	11.00	93 000	5.16	108 000	2.24
79 000	10.38	94 000	5.00	109 000	2.22
80 000	9.98	95 000	4.42	110 000	2.06

TABLE 10-6.- DECEMBER MEAN VALUES OF $N \cdot 10^6$ AT CAPE CANAVERAL

H , ft	$N \cdot 10^6$	H , ft	$N \cdot 10^6$	H , ft	$N \cdot 10^6$
0	338.81	22 000	137.72	44 000	60.64
1 000	322.24	23 000	132.98	45 000	57.89
2 000	308.78	24 000	128.36	46 000	55.31
3 000	294.24	25 000	123.96	47 000	52.84
4 000	280.52	26 000	119.73	48 000	50.43
5 000	267.25	27 000	115.60	49 000	48.15
6 000	254.64	28 000	111.60	50 000	45.98
7 000	242.93	29 000	107.77	51 000	43.85
8 000	232.64	30 000	104.01	52 000	41.82
9 000	222.54	31 000	100.39	53 000	39.88
10 000	213.17	32 000	96.91	54 000	37.95
11 000	204.94	33 000	93.47	55 000	36.10
12 000	197.12	34 000	90.16	56 000	34.30
13 000	189.89	35 000	86.95	57 000	32.60
14 000	183.07	36 000	83.78	58 000	30.87
15 000	176.48	37 000	80.71	59 000	29.30
16 000	170.27	38 000	77.73	60 000	27.76
17 000	164.38	39 000	74.78	61 000	26.24
18 000	158.63	40 000	71.87	62 000	24.82
19 000	153.20	41 000	69.00	63 000	23.45
20 000	147.86	42 000	66.20	64 000	22.17
21 000	142.69	43 000	63.39	65 000	20.96

TABLE 10-6.- Concluded

H, ft	N • 10 ⁶	H, ft	N • 10 ⁶	H, ft	N • 10 ⁶
66 000	19.81	81 000	8.95	96 000	4.14
67 000	18.74	82 000	8.41	97 000	3.99
68 000	17.72	83 000	8.16	98 000	3.91
69 000	16.78	84 000	7.53	99 000	3.38
70 000	15.96	85 000	7.38	100 000	3.34
71 000	15.19	86 000	6.91	101 000	3.20
72 000	14.39	87 000	6.41	102 000	3.00
73 000	13.48	88 000	6.36	103 000	3.00
74 000	12.86	89 000	5.95	104 000	2.91
75 000	12.36	90 000	5.45	105 000	2.42
76 000	11.46	91 000	5.37	106 000	2.30
77 000	11.10	92 000	5.11	107 000	2.27
78 000	10.40	93 000	4.93	108 000	2.17
79 000	9.91	94 000	4.39	109 000	2.01
80 000	9.40	95 000	4.36	110 000	2.00

TABLE 10-7.- VALUES OF $\Delta\rho_7$ AND ΔE_{18} FOR $N_O = 0.00035589$
AND $H_{S1} = 6167$ METERS, $H_{S2} = 6537$ METERS, $H_{S3} = 6652$ METERS

H , meters	E_{Mi} , deg	H_S , meters	$\Delta\rho_7$, meters	ΔE_{18} , mrad
1.E6	1	Table 10-4	74.0	9.92
1.E6	1	6167	69.0	10.21
1.E6	1	6537	71.3	9.97
1.E6	1	6652	72.0	9.89
1.E6	3	Table 10-4	39.2	5.39
1.E6	3	6167	35.2	5.51
1.E6	3	6537	36.9	5.45
1.E6	3	6652	37.5	5.43
1.E6	5	Table 10-4	26.0	3.62
1.E6	5	6167	23.0	3.67
1.E6	5	6537	24.3	3.65
1.E6	5	6652	24.7	3.64
1.E6	10	Table 10-4	13.8	1.94
1.E6	10	6167	12.1	1.95
1.E6	10	6537	12.9	1.94
1.E6	10	6652	13.1	1.94
1.E4	1	Table 10-4	62.0	5.72
1.E4	1	6167	60.0	5.93
1.E4	1	6537	61.2	5.67
1.E4	1	6652	61.6	5.59
1.E4	3	Table 10-4	31.4	2.85
1.E4	3	6167	29.8	3.01
1.E4	3	6537	30.7	2.90
1.E4	3	6652	31.0	2.87
1.E4	5	Table 10-4	20.3	1.84
1.E4	5	6167	19.2	1.95
1.E4	5	6537	19.9	1.88
1.E4	5	6652	20.1	1.86
1.E4	10	Table 10-4	10.6	.95
1.E4	10	6167	10.0	1.00
1.E4	10	6537	10.4	.97
1.E4	10	6652	10.5	.96

11.0 REFERENCES

1. Bean, B. R.; Thayer, B. D.; and Cahoon, B. A.: Methods of Predicting the Atmospheric Bending of Radio Waves. NES Report 6056, National Bureau of Standards, Boulder, Colorado, May 18, 1959.
2. Ward, James: Personal communication, Aug. 1979, Patrick Air Force Base.
3. Lear, William M.: Accuracy and Speed of 38 Self-Starting Integrators. JSC IN 78-FM-39, June 1978.
4. Lear, William M.: Program SEARCH. JSC IN 78-FM-10, Feb. 1978.
5. Station Characteristics for Apollo Mission Support. Flight Software Branch/Flight Support Division, NASA/JSC, May 1967.

APPENDIX A

This appendix contains tables of refraction corrections for

$N_0 = 0.000395$

$H_S = 5446$ meters

$R_0 = 6378165$ meters

$h_i = 0$ unless otherwise specified

The tables A-1 through A-40 were generated by using the set of three nonlinear differential equations to obtain the ray path. An integration step size of 5000 meters was used. The integrator was a fourth-order Runge-Kutta-Gill integrator.

The following definitions are used

N_0 = modulus of refraction at zero altitude above R_0

H_S = atmospheric scale height

R_0 = radius of Earth, value is not critical

h_i = initial altitude of ray path above R_0

h_f = final altitude of ray path above R_0

E_{Mi} = initial measured elevation angle of ray path

E_{Mf} = final elevation angle of ray path

ρ_M = measured range = $\int n ds$

ρ = geometric, straight-line, value of range

E = geometric elevation angle

Figures A-1 and A-2 show the refraction corrections for range and elevation angle for an altitude of $h_f = 10^8$ meters.

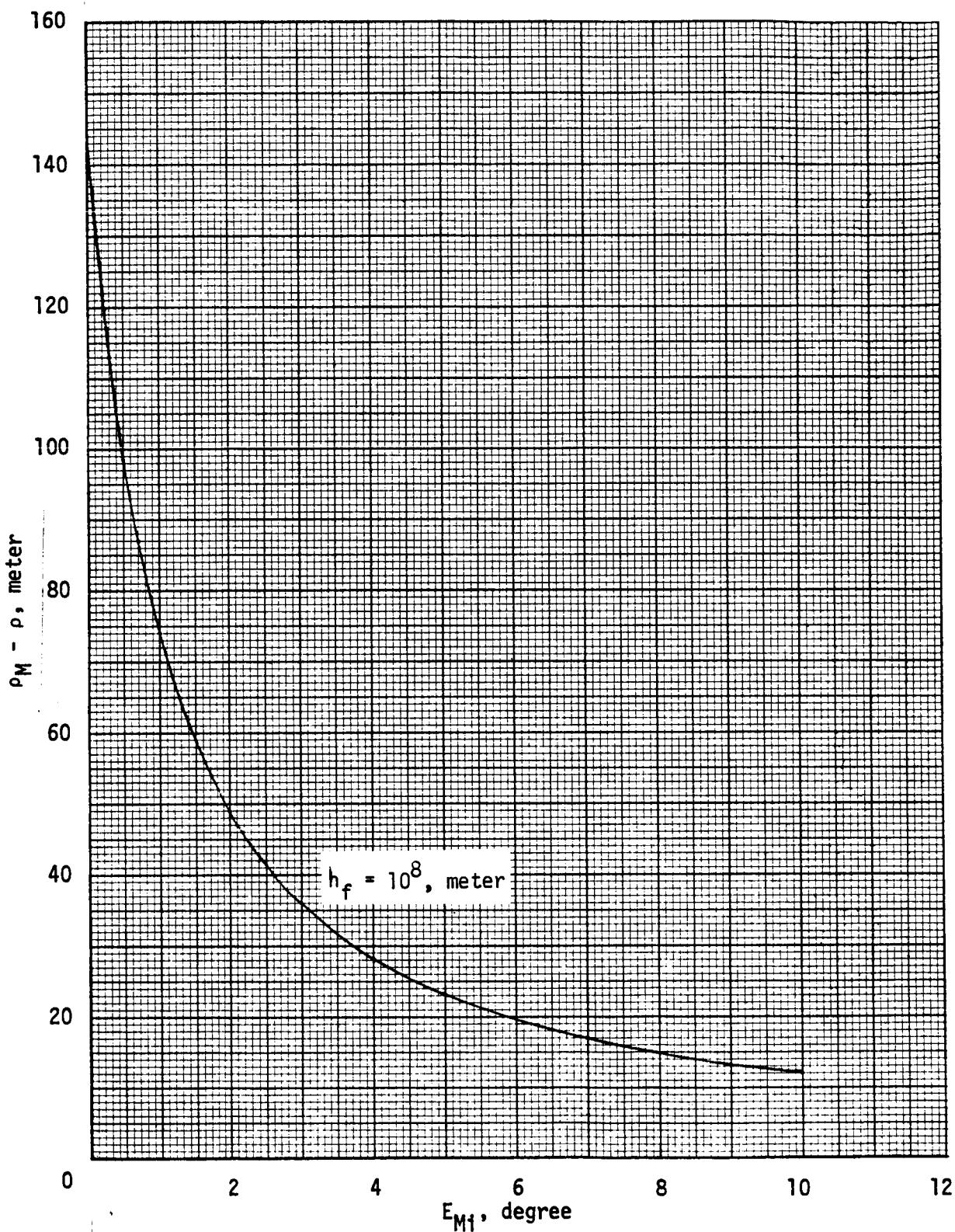


Figure A-1.- Refraction correction for range.

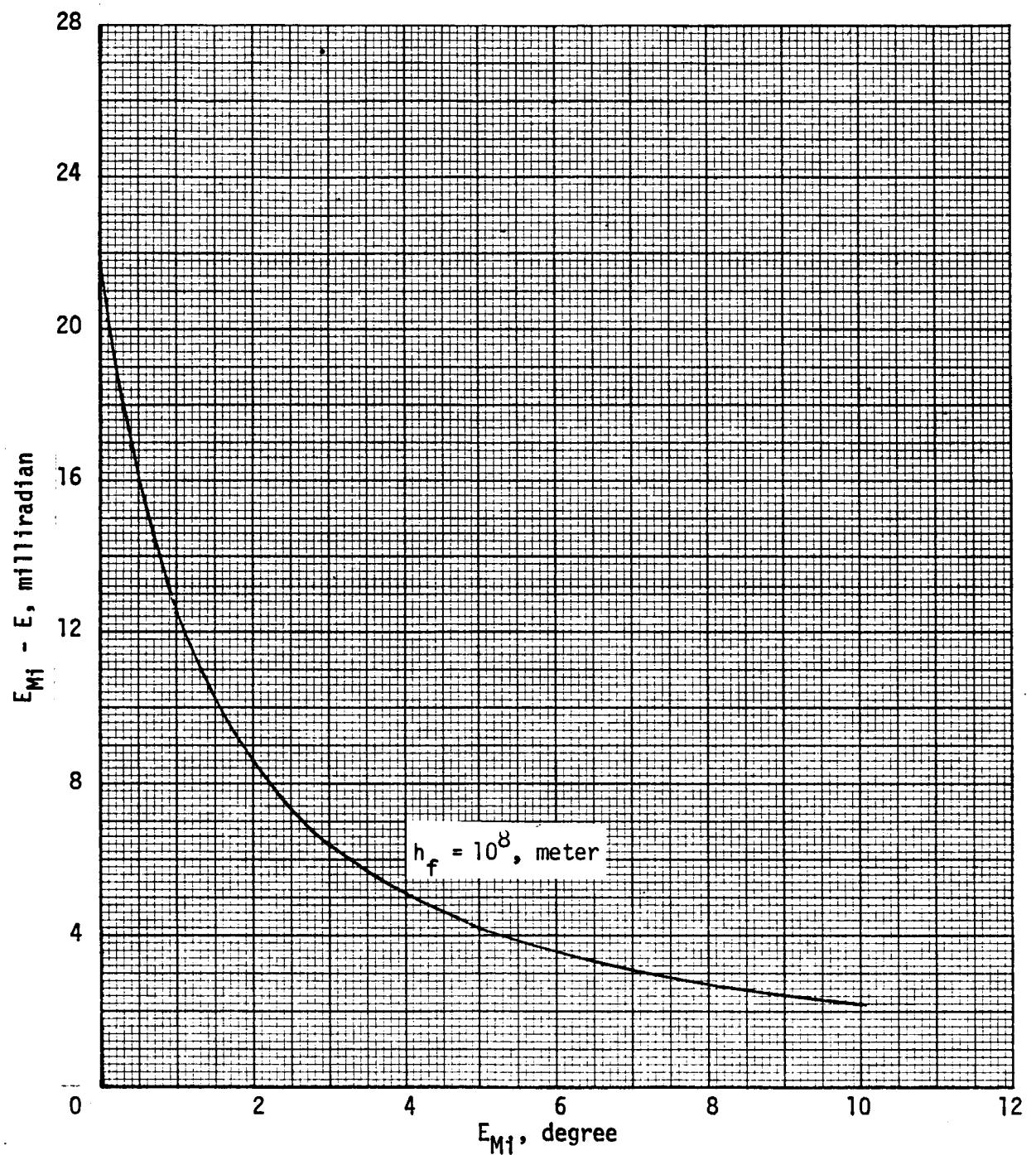


Figure A-2.- Refraction correction for elevation angle.

TABLE A-1.- $\rho_M = 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.0421	999.61	-0.00208	0.39	0.036
.1	1.79	999.61	.09792	.39	.036
.2	3.53	999.61	.19792	.39	.036
.3	5.28	999.61	.29792	.39	.036
.4	7.02	999.61	.39792	.39	.036
.5	8.77	999.61	.49792	.39	.036
.7	12.25	999.61	.69793	.39	.036
1	17.49	999.61	.99793	.39	.036
2	34.93	999.61	1.99793	.39	.036
3	52.36	999.61	2.99793	.39	.036
4	69.77	999.61	3.99794	.39	.036
5	87.16	999.61	4.99794	.39	.036
7	121.86	999.61	6.99795	.39	.036
10	173.62	999.61	9.99798	.39	.035
20	341.93	999.62	19.99809	.38	.033
30	499.84	999.62	29.99826	.38	.030
40	642.57	999.63	39.99847	.37	.027
50	765.78	999.63	49.99873	.37	.022
70	939.36	999.64	69.99933	.36	.012
90	999.64	999.64	90	.361	0

TABLE A-2.- $\rho_M = 2 \cdot 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.168	1999.21	-0.00415	0.79	0.072
.1	3.66	1999.21	.09585	.79	.072
.2	7.15	1999.21	.19585	.79	.072
.3	10.64	1999.21	.29585	.79	.072
.4	14.13	1999.21	.39585	.79	.072
.5	17.61	1999.21	.49585	.79	.072
.7	24.59	1999.21	.69585	.79	.072
1	35.06	1999.21	.99586	.79	.072
2	69.94	1999.22	1.99587	.78	.072
3	104.80	1999.22	2.99588	.78	.072
4	139.63	1999.22	3.99589	.78	.072
5	174.41	1999.22	4.99591	.78	.071
7	243.81	1999.23	6.99594	.77	.071
10	347.33	1999.23	9.99600	.77	.070
20	683.94	1999.26	19.99626	.74	.065
30	999.77	1999.28	29.99661	.72	.059
40	1285.23	1999.30	39.99705	.70	.051
50	1531.64	1999.31	49.99756	.69	.043
70	1878.78	1999.33	69.99873	.67	.022
90	1999.4	1999.34	90	.661	0

TABLE A-3.- $\rho_M = 5 \cdot 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1.05	4998.03	-0.01038	1.97	0.181
.1	9.78	4998.03	.08962	1.97	.181
.2	18.50	4998.03	.18963	1.97	.181
.3	27.22	4998.03	.28964	1.97	.181
.4	35.95	4998.03	.38964	1.97	.181
.5	44.67	4998.03	.48965	1.97	.181
.7	62.12	4998.04	.68966	1.96	.180
1	88.29	4998.04	.98968	1.96	.180
2	175.49	4998.06	1.98974	1.94	.179
3	262.64	4998.07	2.98980	1.93	.178
4	349.72	4998.09	3.98986	1.91	.177
5	436.68	4998.10	4.98993	1.90	.176
7	610.19	4998.13	6.99007	1.87	.173
10	868.99	4998.18	9.99030	1.82	.169
20	1710.53	4998.31	19.99119	1.69	.154
30	2500.09	4998.42	29.99224	1.58	.135
40	3213.69	4998.51	39.99340	1.49	.115
50	3829.64	4998.58	49.99465	1.42	.093
70	4697.37	4998.68	69.99728	1.32	.047
90	4998.71	4998.71	90	1.292	0

TABLE A-4.- $\rho_M = 10^4$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	4.21	9996.05	-0.02076	3.95	0.362
.1	21.66	9996.06	.07926	3.94	.362
.2	39.11	9996.06	.17929	3.94	.362
.3	56.56	9996.07	.27931	3.93	.361
.4	74.01	9996.08	.37933	3.92	.361
.5	91.46	9996.08	.47935	3.92	.360
.7	126.36	9996.10	.67940	3.90	.360
1	178.70	9996.11	.97946	3.89	.358
2	353.14	9996.18	1.97969	3.82	.354
3	527.48	9996.24	2.97992	3.76	.351
4	701.65	9996.29	3.98015	3.71	.346
5	875.60	9996.35	4.98038	3.65	.342
7	1222.66	9996.46	6.98085	3.54	.334
10	1740.32	9996.62	9.98156	3.38	.322
20	3423.49	9997.07	19.98400	2.93	.279
30	5002.53	9997.42	29.98646	2.58	.236
40	6429.49	9997.68	39.98886	2.32	.194
50	7661.07	9997.88	49.99120	2.12	.154
70	9395.82	9998.12	69.99569	1.88	.075
90	9998.19	9998.19	90	1.809	0

TABLE A-5.- $\rho_M = 2 \cdot 10^4$ METERS

E_{Mi} , deg	hf, meters	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	16.85	19 992.11	-0.04150	7.89	0.724
.1	51.77	19 992.13	.05859	7.87	.723
.2	86.70	19 992.16	.15867	7.84	.721
.3	121.62	19 992.18	.25876	7.82	.720
.4	156.54	19 992.21	.35885	7.79	.718
.5	191.47	19 992.23	.45894	7.77	.717
.7	261.31	19 992.28	.65911	7.72	.714
1	366.06	19 992.36	.95938	7.64	.709
2	715.16	19 992.59	1.96024	7.41	.694
3	1064.03	19 992.82	2.96108	7.18	.679
4	1412.57	19 993.04	3.96192	6.96	.665
5	1760.66	19 993.25	4.96273	6.75	.650
7	2455.12	19 993.64	6.96432	6.36	.623
10	3490.85	19 994.16	9.96658	5.84	.583
20	6857.75	19 995.51	19.97325	4.49	.467
30	10 015.26	19 996.39	29.97876	3.61	.371
40	12 867.73	19 996.97	39.98339	3.03	.290
50	15 328.96	19 997.36	49.98736	2.64	.221
70	18 794.72	19 997.78	69.99408	2.22	.103
90	19 997.90	19 997.90	90	2.097	0

TABLE A-6 .- $\rho_M = 5 \cdot 10^4$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	105.56	49 980.36	-0.10348	19.64	1.806
.1	193.27	49 980.51	-.00293	19.49	1.796
.2	280.98	49 980.67	.09762	19.33	1.787
.3	368.68	49 980.82	.19816	19.18	1.777
.4	456.38	49 980.97	.29870	19.03	1.768
.5	544.07	49 981.12	.39923	18.88	1.759
.7	719.44	49 981.42	.60029	18.58	1.740
1	982.45	49 981.85	.90184	18.15	1.713
2	1858.69	49 983.19	1.90678	16.81	1.627
3	2734.04	49 984.40	2.91138	15.60	1.547
4	3608.25	49 985.49	3.91566	14.51	1.472
5	4481.07	49 986.48	4.91965	13.52	1.402
7	6221.53	49 988.17	6.92685	11.83	1.277
10	8815.52	49 990.17	9.93600	9.83	1.117
20	17 236.22	49 994.00	19.95686	6.00	.753
30	25 121.15	49 995.75	29.96930	4.25	.536
40	32 236.76	49 996.66	39.97761	3.34	.391
50	38 371.48	49 997.20	49.98372	2.80	.284
70	47 003.08	49 997.71	69.99272	2.29	.127
90	49 997.85	49 997.85	90	2.151	0

TABLE A-7.- $\rho_M = 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	425.69	99 961.31	-0.20498	38.69	3.578
.1	603.92	99 961.92	-.10281	38.08	3.540
.2	782.09	99 962.52	-.00069	37.48	3.503
.3	960.20	99 963.11	.10141	36.89	3.466
.4	1138.25	99 963.69	.20347	36.31	3.430
.5	1316.24	99 964.25	.30550	35.75	3.395
.7	1672.04	99 965.34	.50947	34.66	3.325
1	2205.29	99 966.90	.81520	33.10	3.225
2	3979.11	99 971.42	1.83251	28.58	2.923
3	5747.39	99 975.11	2.84744	24.89	2.663
4	7510.24	99 978.14	3.86039	21.86	2.437
5	9267.68	99 980.64	4.87167	19.36	2.240
7	12 765.84	99 984.46	6.89026	15.54	1.915
10	17 968.05	99 988.26	9.91090	11.74	1.555
20	34 803.29	99 993.75	19.94790	6.25	.909
30	50 528.87	99 995.71	29.96515	4.29	.608
40	64 699.37	99 996.66	39.97535	3.34	.430
50	76 902.79	99 997.19	49.98238	2.81	.308
70	94 052.91	99 997.71	69.99225	2.29	.135
90	99 997.85	99 997.85	90	2.152	0

TABLE A-8.- $\rho_M = 2 \cdot 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1755.62	199 927.18	-0.39479	72.82	6.890
.1	2132.93	199 929.47	-.28664	70.53	6.748
.2	2509.32	199 931.66	-.17875	68.34	6.610
.3	2884.84	199 933.76	-.07111	66.24	6.477
.4	3259.50	199 935.76	.03629	64.24	6.348
.5	3633.34	199 937.68	.14346	62.32	6.223
.7	4378.68	199 941.27	.35715	58.73	5.984
1	5491.25	199 946.11	.67617	53.89	5.652
2	9159.93	199 958.55	1.72869	41.45	4.735
3	12 780.38	199 967.03	2.76850	32.97	4.041
4	16 364.49	199 973.00	3.79932	27.00	3.502
5	19 919.88	199 977.34	4.82367	22.66	3.078
7	26 962.31	199 983.06	6.85925	16.94	2.457
10	37 386.00	199 987.84	9.89317	12.16	1.864
20	70 955.85	199 993.74	19.94304	6.26	.994
30	102 202.79	199 995.70	29.96303	4.30	.645
40	130 291.22	199 996.6	39.97421	3.34	.450
50	154 431.70	199 997.19	49.98171	2.81	.319
70	188 283.39	199 997.71	69.99201	2.29	.140
90	199 997.85	199 997.85	90	2.152	0

TABLE A-9.- $\rho_M = 5 \cdot 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	12 710 .28	499 874 .76	-0 .78693	125 .24	13 .735
.1	13 881 .39	499 883 .03	-.65247	116 .97	13 .133
.2	15 029 .82	499 890 .41	-.52060	109 .59	12 .577
.3	16 157 .94	499 897 .02	-.39103	102 .98	12 .061
.4	17 267 .76	499 902 .97	-.26354	97 .03	11 .581
.5	18 361 .07	499 908 .35	-.13793	91 .65	11 .134
.7	20 504 .12	499 917 .68	.10835	82 .32	10 .326
1	23 627 .88	499 928 .88	.46750	71 .12	9 .294
2	33 509 .72	499 952 .01	1 .60504	47 .99	6 .893
3	42 909 .82	499 964 .33	2 .68936	35 .67	5 .422
4	52 044 .69	499 971 .82	3 .74557	28 .18	4 .441
5	61 014 .07	499 976 .80	4 .78535	23 .20	3 .746
7	78 636 .77	499 982 .94	6 .83748	17 .06	2 .836
10	104 546 .69	499 987 .82	9 .88184	12 .18	2 .062
20	187 362 .49	499 993 .74	19 .94012	6 .26	1 .045
30	263 849 .16	499 995 .70	29 .96176	4 .30	.667
40	332 163 .29	499 996 .66	39 .97353	3 .34	.462
50	390 555 .57	499 997 .19	49 .98130	2 .81	.326
70	471 956 .44	499 997 .71	69 .99186	2 .29	.142
90	499 997 .85	499 997 .85	90	2 .152	0

TABLE A-10.- $\rho_M = 10^6$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	60 411.21	999 863.91	-1.01283	136.09	17.677
.1	63 127.55	999 874.36	-.85573	125.64	16.681
.2	65 744.21	999 883.41	-.70433	116.59	15.784
.3	68 275.48	999 891.32	-.55782	108.68	14.972
.4	70 732.93	999 898.30	-.41552	101.70	14.233
.5	73 126.06	999 904.49	-.27689	95.51	13.559
.7	77 749.63	999 915.00	-.00892	85.00	12.373
1	84 363.25	999 927.26	.37475	72.74	10.913
2	104 691.86	999 951.61	1.55678	48.39	7.736
3	123 606.74	999 964.20	2.66059	35.80	5.924
4	141 808.21	999 971.77	3.72676	28.23	4.769
5	159 585.39	999 976.78	4.77224	23.22	3.975
7	194 353.43	999 982.93	6.83017	17.07	2.964
10	245 226.14	999 987.82	9.87806	12.18	2.128
20	406 460.06	999 993.74	19.93915	6.26	1.062
30	553 765.54	999 995.70	29.96133	4.30	.675
40	684 130.05	999 996.66	39.97330	3.34	.466
50	794 712.79	999 997.19	49.98117	2.81	.329
70	947 636.16	999 997.71	69.99182	2.29	.143
90	999 997.85	999 997.85	90	2.152	0

TABLE A-11.- $\rho_M = 2 \cdot 10^6$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	268 503.6	1 999 859.9	-1.1280	140.1	19.69
.1	274 163.5	1 999 871.1	-.9591	128.9	18.48
.2	279 570.5	1 999 880.8	-.7975	119.2	17.41
.3	284 763.3	1 999 889.2	-.6422	110.8	16.44
.4	289 772.8	1 999 896.5	-.4923	103.5	15.57
.5	294 624.2	1 999 990.3	-.3470	97.0	14.78
.7	303 931.1	1 999 913.9	-.0679	86.1	13.40
1	317 119.3	1 999 926.6	.3282	73.4	11.73
2	357 051.8	1 999 951.4	1.5326	48.6	8.16
3	393 740.4	1 999 964.1	2.6462	35.9	6.18
4	428 813.7	1 999 971.7	3.7174	28.3	4.93
5	462 923.1	1 999 976.8	4.7657	23.2	4.09
7	529 324.9	1 999 982.9	6.8265	17.1	3.03
10	625 893.0	1 999 987.8	9.8762	12.2	2.16
20	928 072.7	1 999 993.7	19.9387	6.3	1.07
30	1 199 583.9	1 999 995.7	29.9611	4.3	.68
40	1 436 630.1	1 999 996.7	39.9732	3.3	.47
50	1 635 534.0	1 999 997.2	49.9811	2.8	.33
70	1 907 591.5	1 999 997.7	69.9918	2.3	.14
90	1 999 997.8	1 999 997.8	90	2.152	0

TABLE A-12.- $\rho_M = 5 \cdot 10^6$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1 643 494.7	4 999 857.4	-1.1972	142.6	20.89
.1	1 655 710.1	4 999 869.2	-1.0210	130.8	19.57
.2	1 667 322.5	4 999 879.2	-.8534	120.8	18.38
.3	1 678 426.6	4 999 887.9	-.6928	112.1	17.33
.4	1 689 097.6	4 999 895.4	-.5383	104.6	16.38
.5	1 699 396.2	4 999 902.1	-.3890	97.9	15.52
.7	1 719 065.1	4 999 913.3	-.1033	86.7	14.02
1	1 746 766.6	4 999 926.2	.3003	73.8	12.21
2	1 829 775.1	4 999 951.3	1.5181	48.7	8.41
3	1 905 294.6	4 999 964.1	2.6376	35.9	6.33
4	1 977 053.9	4 999 971.7	3.7117	28.3	5.03
5	2 046 525.2	4 999 976.8	4.7617	23.2	4.16
7	2 181 021.5	4 999 982.9	6.8243	17.1	3.07
10	2 375 085.0	4 999 987.8	9.8750	12.2	2.18
20	2 972 290.9	4 999 993.7	19.9384	6.3	1.08
30	3 497 772.0	4 999 995.7	29.9610	4.3	.68
40	3 949 276.9	4 999 996.7	39.9731	3.3	.47
50	4 323 561.8	4 999 997.2	49.9811	2.8	.33
70	4 829 553.5	4 999 997.7	69.9918	2.3	.14
90	4 999 997.8	4 999 997.8	90	2.152	0

TABLE A-13.- $\rho_M = 10^7$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	5 367 545.9	9 999 856.6	-1.2202	143.4	21.30
.1	5 384 456.9	9 999 868.5	-1.0417	131.5	19.93
.2	5 400 512.7	9 999 878.7	-.8720	121.3	18.71
.3	5 415 848.7	9 999 887.4	-.7097	112.6	17.62
.4	5 430 572.4	9 999 895.1	-.5536	104.9	16.64
.5	5 444 770.0	9 999 901.8	-.4030	98.2	15.76
.7	5 471 856.6	9 999 913.1	-.1151	86.9	14.23
1	5 509 951.6	9 999 926.1	.2910	73.9	12.37
2	5 623 869.8	9 999 951.3	1.5133	48.7	8.49
3	5 727 372.4	9 999 964.1	2.6347	35.9	6.38
4	5 825 696.0	9 999 971.7	3.7098	28.3	5.06
5	5 920 897.4	9 999 976.8	4.7604	23.2	4.18
7	6 105 294.8	9 999 982.9	6.8236	17.1	3.08
10	6 371 618.5	9 999 987.8	9.8747	12.2	2.19
20	7 193 160.8	9 999 993.7	19.9383	6.3	1.08
30	7 918 240.9	9 999 995.7	29.9610	4.3	.68
40	8 542 663.1	9 999 996.7	39.9731	3.3	.47
50	9 061 165.7	9 999 997.2	49.9810	2.8	.33
70	9 763 239.6	9 999 997.7	69.9918	2.3	.14
90	9 999 997.8	9 999 997.8	90	2.152	0

TABLE A-14.- $\rho_M = 2 \cdot 10^7$ METERS

E_{M1} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{M1} - E$, mrad
0	14 483 070.0	19 999 856.2	-1.2317	143.8	21.50
.1	14 502 245.1	19 999 868.2	-1.0520	131.8	20.11
.2	14 520 447.6	19 999 878.4	-.8813	121.6	18.87
.3	14 537 832.3	19 999 887.2	-.7181	112.8	17.77
.4	14 554 421.9	19 999 894.9	-.5613	105.1	16.78
.5	14 570 615.1	19 999 901.7	-.4100	98.3	15.88
.7	14 601 319.3	19 999 913.0	-.1210	87.0	14.33
1	14 644 511.3	19 999 926.0	.2863	74.0	12.46
2	14 773 807.9	19 999 951.3	1.5109	48.7	8.54
3	14 891 548.3	19 999 964.1	2.6332	35.9	6.40
4	15 003 680.2	19 999 971.7	3.7089	28.3	5.08
5	15 112 530.6	19 999 976.8	4.7598	23.2	4.19
7	15 324 161.1	19 999 982.9	6.8232	17.1	3.09
10	15 631 654.8	19 999 987.8	9.8745	12.2	2.19
20	16 593 134.6	19 999 993.7	19.9382	6.3	1.08
30	17 456 545.0	19 999 995.7	29.9609	4.3	.68
40	18 210 068.4	19 999 996.7	39.9731	3.3	.47
50	18 842 161.8	19 999 997.2	49.9810	2.8	.33
70	19 706 484.1	19 999 997.7	69.9918	2.3	.14
90	19 999 997.8	19 999 997.8	90	2.152	0

TABLE A-15.- $\rho_M = 5 \cdot 10^7$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	43 889 909.6	49 999 856.0	-1.2386	144.0	21.62
.1	43 909 888.0	49 999 868.0	-1.0582	132.0	20.21
.2	43 928 855.2	49 999 878.2	-.8869	121.8	18.97
.3	43 946 972.8	49 999 887.1	-.7232	112.9	17.86
.4	43 964 368.6	49 999 894.8	-.5659	105.2	16.86
.5	43 981 145.7	49 999 901.6	-.4142	98.4	15.96
.7	44 013 163.8	49 999 912.9	-.1245	87.1	14.39
1	44 058 227.9	49 999 926.0	.2835	74.0	12.50
2	44 193 342.0	49 999 951.3	1.5094	48.7	8.56
3	44 316 713.5	49 999 964.1	2.6324	35.9	6.42
4	44 434 535.6	49 999 971.9	3.7083	28.3	5.09
5	44 549 226.2	49 999 976.7	4.7594	23.3	4.20
7	44 773 115.7	49 999 982.9	6.8230	17.1	3.09
10	45 100 546.4	49 999 987.8	9.8744	12.2	2.19
20	46 140 210.5	49 999 993.7	19.9382	6.3	1.08
30	47 093 434.3	49 999 995.7	29.9609	4.3	.68
40	47 939 710.1	49 999 996.7	39.9731	3.3	.47
50	48 659 465.2	49 999 997.2	49.9810	2.8	.33
70	49 657 545.5	49 999 997.7	69.9918	2.3	.14
90	49 999 997.8	49 999 997.8	90	2.154	0

TABLE A-16.- $\rho_M = 10^8$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	93 686 945.2	99 999 855.9	-1.2409	144.1	21.66
.1	93 707 046.3	99 999 867.9	-1.0603	132.1	20.25
.2	93 726 131.0	99 999 878.2	-.8888	121.8	19.00
.3	93 744 362.0	99 999 887.0	-.7249	113.0	17.89
.4	93 761 868.0	99 999 894.7	-.5675	105.3	16.89
.5	93 778 752.6	99 999 901.5	-.4156	98.5	15.98
.7	93 810 980.0	99 999 912.9	-.1257	87.1	14.41
1	93 856 348.8	99 999 925.9	.2826	74.1	12.52
2	93 992 462.7	99 999 951.3	1.5090	48.8	8.57
3	94 116 879.8	99 999 964.1	2.6321	35.9	6.42
4	94 235 829.8	99 999 971.7	3.7082	28.3	5.09
5	94 351 743.6	99 999 976.7	4.7593	23.3	4.20
7	94 578 382.4	99 999 982.9	6.8229	17.1	3.09
10	94 910 697.5	99 999 987.8	9.8743	12.2	2.19
20	95 972 596.4	99 999 993.7	19.9382	6.3	1.08
30	96 954 999.2	99 999 995.7	29.9609	4.3	.68
40	97 834 032.6	99 999 996.7	39.9731	3.4	.47
50	98 586 593.4	99 999 997.2	49.9810	2.8	.33
70	99 637 497.7	99 999 997.7	69.9918	2.3	.14
90	99 999 997.8	99 999 997.8	90	2.162	0

TABLE A-17 .- $h_f = 10$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.074418	15 402.36	-0.03198	6.08	.558
.1	.124652	5101.00	.08941	2.10	.185
.2	.213396	2771.98	.19424	1.09	.100
.3	.309092	1881.37	.29609	.74	.068
.4	.406864	1420.22	.39705	.56	.051
.5	.505508	1139.66	.49763	.45	.041
.7	.703944	816.23	.69831	.32	.030
1	1.002765	572.20	.99881	.23	.021
2	2.001383	286.44	1.99941	.11	.010
3	3.000922	191.04	2.99960	.08	.007
4	4.000691	143.34	3.99970	.06	.005
5	5.000552	114.73	4.99976	.05	.004
7	7.000394	82.05	6.99983	.03	.003
10	10.000274	57.59	9.99988	.02	.002
20	20.000133	29.24	19.99994	.01	.001
30	30.000084	20.00	29.99996	.01	.001
40	40.000058	15.56	39.99998	.01	0
50	50.000041	13.05	49.99998	.01	0
70	70.000018	10.64	70.00000	0	0
90	90	10	90	0	0

TABLE A-18.- $h_f = 20$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.105285	21 779.38	-0.04521	8.59	0.789
.1	.145206	9347.46	.08061	3.69	.338
.2	.226019	5379.82	.18884	2.12	.195
.3	.317938	3708.91	.29231	1.46	.134
.4	.413624	2816.87	.39416	1.11	.102
.5	.510964	2267.02	.49530	.89	.082
.7	.707873	1627.91	.69662	.64	.059
1	1.005527	1142.82	.99763	.45	.041
2	2.002768	572.68	1.9881	.23	.021
3	3.001845	382.03	2.99921	.15	.014
4	4.001383	286.66	3.99941	.11	.010
5	5.001106	229.45	4.99953	.09	.008
7	7.000788	164.10	6.99966	.06	.006
10	10.000549	115.17	9.99976	.05	.004
20	20.000168	58.48	19.99989	.02	.002
30	30.000168	40.00	29.99993	.02	.001
40	40.000115	31.11	39.99995	.01	.001
50	50.000081	26.11	49.99997	.01	.001
70	70.000035	21.28	69.99998	.01	0
90	90	20	90	.01	0

TABLE A-19 .- $h_f = 50$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.166665	34 422.73	-0.07139	13.56	1.246
.1	.194364	19 472.45	.05966	7.66	.704
.2	.260340	12 448.55	.17422	4.90	.450
.3	.343186	8908.94	.28155	3.50	.322
.4	.433332	6875.92	.38576	2.70	.249
.5	.527045	5578.97	.48845	2.19	.202
.7	.719566	4036.32	.69164	1.59	.146
1	1.013792	2845.34	.99411	1.12	.103
2	2.006929	1430.21	1.99704	.56	.052
3	3.004622	954.63	2.99803	.38	.034
4	4.003465	716.47	3.99852	.28	.026
5	5.002770	573.53	4.99882	.23	.021
7	7.001974	410.22	6.99916	.16	.015
10	10.001375	287.92	9.99941	.11	.010
20	20.000666	146.19	19.99972	.06	.005
30	30.000420	100.00	29.99982	.04	.003
40	40.000289	77.79	39.99988	.03	.002
50	50.000203	65.27	49.99991	.03	.002
70	70.000088	53.21	69.99996	.02	.001
90	90	50	90	.02	0

TABLE A-20.- $h_f = 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.236157	48 649.47	-0.10074	19 .12	1.758
.1	.256457	32 183.93	.03347	12 .62	1.161
.2	.309467	22 504.98	.15351	8 .82	.811
.3	.381798	16 812.56	.26528	6 .58	.606
.4	.464510	13 257.77	.37263	5 .19	.478
.5	.552964	10 884.33	.47753	4 .26	.392
.7	.738760	7965.36	.68356	3 .12	.287
1	1.027504	5652.35	.98833	2 .21	.204
2	2.013889	2855.49	1.99411	1 .12	.103
3	3.009272	1907.79	2.99607	.75	.069
4	4.006954	1432.32	3.99705	.56	.051
5	5.005560	1146.74	4.99764	.45	.041
7	7.003963	820.32	6.99832	.32	.029
10	10.002760	575.80	9.99883	.23	.020
20	20.001337	292.37	19.99943	.11	.010
30	30.000843	200.00	29.99964	.08	.006
40	40.000580	155.57	39.99975	.06	.004
50	50.000480	130.54	49.99983	.05	.003
70	70.000177	106.42	69.99992	.04	.001
90	90	100	90	.04	0

TABLE A-21.- $h_f = 2 \cdot 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.335256	68 712.25	-0.14185	26.88	2.476
.1	.349852	51 091.12	-.00519	19.91	1.836
.2	.390380	38 883.61	.12006	15.12	1.395
.3	.449884	30 593.78	.23715	11.89	1.097
.4	.521915	24 876.46	.34892	9.66	.892
.5	.601992	20 807.28	.45728	8.08	.746
.7	.776139	15 530.35	.66813	6.03	.556
1	1.054697	11 156.21	.97711	4.33	.400
2	2.027893	5691.27	1.98833	2.21	.204
3	3.018658	3809.69	2.99219	1.48	.136
4	4.014002	2862.14	3.99414	1.11	.102
5	5.011199	2292.20	4.99531	.89	.082
7	7.007984	1640.18	6.99666	.64	.058
10	10.005561	1151.44	9.99767	.45	.041
20	20.002695	584.72	19.99887	.23	.020
30	30.001699	399.99	29.99929	.16	.012
40	40.001169	311.14	39.99951	.12	.009
50	50.000823	261.08	49.99966	.10	.006
70	70.000357	212.84	69.99985	.08	.003
90	90	200	90	.08	0

TABLE A-22.- $h_f = 5 \cdot 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.535963	108 235.50	-0.22145	41.75	3.865
.1	.545212	89 558.25	-.08236	34.31	3.183
.2	.572062	74 650.42	.04848	28.47	2.645
.3	.614210	62 936.94	.17253	23.93	2.225
.4	.668769	53 774.03	.29124	20.40	1.898
.5	.732973	46 577.20	.40589	17.65	1.643
.7	.881613	36 277.73	.62678	13.73	1.278
1	1.134560	26 863.99	.94583	10.15	.945
2	2.070541	14 081.62	1.97164	5.32	.495
3	3.047457	9479.66	2.98092	3.58	.333
4	4.035689	7136.39	3.98566	2.69	.250
5	5.028571	5720.75	4.98852	2.16	.200
7	7.020386	4096.89	6.99181	1.55	.143
10	10.014207	2877.39	9.99429	1.09	.100
20	20.006886	1461.66	19.99723	.55	.048
30	30.004342	999.94	29.99826	.38	.030
40	40.002987	777.84	39.99880	.29	.021
50	50.002103	652.69	49.99915	.25	.015
70	70.000912	532.09	69.99963	.20	.006
90	90	500	90	.19	0

TABLE A-23 .- $h_f = 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.770977	152 157.31	-0.30684	57.34	5.355
.1	.777435	132 956.71	-.16621	49.60	4.646
.2	.796494	116 588.13	-.03219	43.16	4.052
.3	.827284	102 745.27	.09621	37.82	3.557
.4	.868559	91 094.84	.21986	33.39	3.144
.5	.918907	81 303.88	.33959	29.71	2.800
.7	1.041333	66 100.47	.56998	24.05	2.269
1	1.262674	50 779.68	.90037	18.41	1.739
2	2.143400	27 683.83	1.94584	10.00	.945
3	3.097396	18 809.10	2.96325	6.79	.641
4	4.073505	14 208.17	3.97228	5.13	.484
5	5.058942	11 408.14	4.97777	4.12	.388
7	7.042119	8181.56	6.98412	2.95	.277
10	10.029375	5750.60	9.98892	2.08	.193
20	20.014247	2922.83	19.99463	1.05	.094
30	30.008983	1999.73	29.99661	.72	.059
40	40.006181	1555.63	39.99767	.56	.041
50	50.004352	1305.37	49.99836	.47	.029
70	70.001888	1064.17	69.99929	.38	.012
90	90	10^3	90	.36	0

TABLE A-24.- $h_f = 2 \cdot 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1.123180	212 844.17	-0.41754	76.70	7.287
.1	1.127622	193 294.44	-.27527	68.62	6.550
.2	1.140845	175 914.64	-.13862	61.70	5.910
.3	1.162550	160 497.96	-.00680	55.74	5.355
.4	1.192272	146 847.54	.12089	50.60	4.871
.5	1.229431	134 774.73	.24503	46.14	4.450
.7	1.323431	114 657.78	.88459	38.87	3.760
1	1.503797	92 265.13	.82779	31.01	3.006
2	2.293692	53 530.58	1.90092	17.79	1.729
3	3.203183	37 010.77	2.93170	12.27	1.192
4	4.154453	28 149.50	3.94815	9.32	.905
5	5.124288	22 677.12	4.95830	7.50	.728
7	7.089094	16 311.71	6.97013	5.40	.521
10	10.062241	11 483.53	9.97914	3.80	.364
20	20.030224	5843.54	19.99899	1.93	.177
30	30.019062	3998.89	29.99361	1.32	.112
40	40.013118	3111.04	39.99560	1.03	.077
50	50.009237	2610.64	49.99690	.86	.054
70	70.004007	2128.33	69.99866	.70	.023
90	90	$2 \cdot 10^3$	90	.66	0

TABLE A-25.- $h_f = 5 \cdot 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1.893878	327 688.12	-0.59726	104.52	10.424
.1	1.896516	307 865.84	-.45191	95.86	9.633
.2	1.904406	289 679.82	-.31178	88.33	8.932
.3	1.917483	272 959.30	-.17607	81.73	8.309
.4	1.935644	257 562.29	-.04415	75.91	7.752
.5	1.958745	243 367.50	.08451	70.73	7.252
.7	2.019058	218 172.52	.33367	61.98	6.394
1	2.141591	187 084.56	.69160	51.84	5.383
2	2.754146	121 842.34	1.80518	32.44	3.400
3	3.547322	88 043.54	2.86084	23.11	2.429
4	4.425036	68 293.97	3.89268	17.82	1.873
5	5.345808	55 577.47	4.91300	14.45	1.519
7	7.250442	40 356.30	6.93720	10.46	1.096
10	10.175966	28 561.40	9.95595	7.39	.769
20	20.085813	14 590.97	19.97855	3.77	.374
30	30.054164	9992.35	29.98646	2.58	.236
40	40.037284	7775.80	39.99068	2.01	.163
50	50.026256	6525.86	49.99344	1.69	.115
70	70.011390	5320.71	69.99715	1.38	.050
90	90	$5 \cdot 10^3$	90	1.29	0

TABLE A-26.- $h_f = 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	2.846580	449 096.93	-0.74036	121.25	12.922
.1	2.848335	429 161.91	-.59151	112.03	12.069
.2	2.853592	410 638.47	-.44803	104.01	11.310
.3	2.862332	393 361.79	-.30910	96.97	10.631
.4	2.874524	377 199.36	-.17404	90.74	10.019
.5	2.890123	362 042.56	-.04232	85.19	9.465
.7	2.931316	334 396.53	.21279	75.74	8.503
1	3.016985	298 586.23	.57930	64.63	7.343
2	3.478466	214 039.27	1.71787	42.46	4.924
3	4.134684	162 802.03	2.79198	31.06	3.631
4	4.908141	129 816.89	3.83683	24.29	2.848
5	5.751733	107 326.43	4.86643	19.86	2.331
7	7.553983	79 182.26	6.90264	14.50	1.699
10	10.393294	56 572.62	9.93132	10.29	1.199
20	20.193344	29 113.60	19.96640	5.27	.586
30	30.122226	19 965.99	29.97878	3.61	.370
40	40.084180	15 544.69	39.98539	2.81	.255
50	50.059295	13 048.85	49.98971	2.36	.180
70	70.025728	10 640.98	69.99553	1.92	.078
90	90	10^4	90	1.81	0

TABLE A-27 .- $h_f = 2 \cdot 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	4.244064	611 129.26	-0.86694	130.33	15.131
.1	4.245240	591 127.69	-.71354	120.53	14.199
.2	4.248765	572 403.87	-.56592	112.03	13.368
.3	4.254634	554 794.55	-.42318	104.58	12.622
.4	4.262838	538 169.85	-.28460	98.00	11.949
.5	4.273362	522 424.66	-.14960	92.15	11.338
.7	4.301300	493 241.93	.11145	82.21	10.272
1	4.360074	454 313.99	.48568	70.54	8.977
2	4.690923	354 636.57	1.64366	47.15	6.219
3	5.195730	285 892.18	2.73143	34.94	4.687
4	5.829476	236 783.72	3.78632	27.56	3.729
5	6.554870	200 679.51	4.82346	22.67	3.081
7	8.180607	152 267.86	6.86999	16.66	2.269
10	10.854895	110 824.36	9.90767	11.88	1.612
20	20.427291	57 924.91	19.95458	6.11	.793
30	30.271018	39 848.36	29.97128	4.19	.501
40	40.186876	31 058.40	39.98022	3.26	.345
50	50.131705	26 084.78	49.98606	2.74	.243
70	70.057173	21 279.97	69.99395	2.23	.106
90	90	$2 \cdot 10^4$	90	2.10	0

TABLE A-28.- $h_f = 5 \cdot 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	6.968210	918 248.68	-0.99235	135.37	17.320
.1	6.968924	898 196.16	-.83234	124.90	16.272
.2	6.971065	879 318.23	-.67879	115.86	15.338
.3	6.974633	861 452.18	-.53070	107.98	14.498
.4	6.979624	844 468.98	-.38727	101.04	13.740
.5	6.986037	828 264.70	-.24786	94.88	13.053
.7	7.003108	797 868.29	.02096	84.47	11.852
1	7.039248	756 412.01	.40472	72.31	10.390
2	7.248186	643 119.83	1.58357	48.19	7.268
3	7.583635	555 475.04	2.68362	35.70	5.522
4	8.029754	485 449.33	3.74669	28.17	4.421
5	8.569277	428 651.66	4.78972	23.19	3.670
7	9.864780	343 567.90	6.84414	17.05	2.720
10	12.168049	261 154.07	9.88876	12.17	1.942
20	21.132101	142 485.70	19.94501	6.26	.960
30	30.725091	98 960.69	29.96519	4.29	.608
40	40.501747	77 399.11	39.97601	3.34	.419
50	50.354196	65 108.51	49.98310	2.81	.295
70	70.153959	53 183.96	69.99266	2.29	.128
90	90	$5 \cdot 10^4$	90	2.15	0

TABLE A-29.- $h_f = 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	9.952149	1 257 985.33	-1.06010	137.74	18.502
.1	9.952647	1 237 909.08	-.89546	126.89	17.374
.2	9.954138	1 218 959.12	-.73781	117.54	16.368
.3	9.956624	1 200 972.86	-.58607	109.40	15.465
.4	9.960103	1 183 821.43	-.43936	102.26	14.650
.5	9.964575	1 167 401.16	-.29700	95.94	13.910
.7	9.976489	1 136 430.70	-.02309	85.26	12.620
1	10.001760	1 093 764.39	.36677	72.85	11.052
2	10.149136	973 619.32	1.55808	48.38	7.713
3	10.390118	875 464.75	2.66468	35.78	5.852
4	10.718395	792 340.33	3.73169	28.22	4.683
5	11.126240	720 924.00	4.77731	23.21	3.887
7	12.147057	605 419.04	6.83494	17.07	2.881
10	14.072509	481 391.89	9.88213	12.18	2.057
20	22.248180	277 765.49	19.94168	6.26	1.018
30	31.461196	195 777.08	29.96307	4.30	.644
40	41.016785	153 981.82	39.97455	3.34	.444
50	50.719665	129 872.13	49.98207	2.81	.313
70	70.313489	106 314.36	69.99222	2.29	.136
90	90	10^5	90	2.15	0

TABLE A-30.- $h_f = 2 \cdot 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	14.074728	1 738 116.0	-1.1107	139.5	19.39
.1	14.075076	1 718 023.4	-.9421	128.4	18.19
.2	14.076120	1 699 023.8	-.7810	118.8	17.12
.3	14.077860	1 680 954.7	-.6262	110.4	16.16
.4	14.080296	1 663 687.2	-.4768	103.1	15.30
.5	14.083427	1 647 117.8	-.3320	96.7	14.52
.7	14.091773	1 615 750.2	-.0539	85.8	13.16
1	14.109492	1 572 243.2	.3409	73.2	11.50
2	14.213275	1 447 245.5	1.5418	48.5	8.00
3	14.384581	1 341 339.2	2.6533	35.8	6.05
4	14.621037	1 248 008.8	3.7230	28.2	4.83
5	14.919545	1 164 454.0	4.7704	23.2	4.01
7	15.687810	1 020 980.5	6.8300	17.1	2.97
10	17.206810	852 191.7	9.8787	12.2	2.12
20	24.288973	530 863.2	19.9400	6.3	1.05
30	32.857052	383 636.8	29.9620	4.3	.66
40	42.008348	304 854.8	39.9738	3.3	.46
50	51.428502	258 411.1	49.9816	2.8	.32
70	70.624820	212 419.1	69.9920	2.3	.14
90	90	2·10 ⁵	90	2.2	0

TABLE A-31.- $h_f = 5 \cdot 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	21.925144	2 706 660.7	-1.1581	141.2	20.21
.1	21.925361	2 686 553.8	-.9855	129.7	18.94
.2	21.926011	2 667 510.7	-.8208	119.9	17.82
.3	21.927095	2 649 368.7	-.6629	111.4	16.81
.4	21.928613	2 631 999.1	-.5107	103.9	15.89
.5	21.930564	2 615 298.4	-.3634	97.4	15.07
.7	21.935765	2 583 580.8	-.0812	86.3	13.64
1	21.946814	2 539 331.2	.3183	73.5	11.90
2	22.011696	2 410 003.2	1.5285	48.6	8.23
3	22.119408	2 297 021.9	2.6443	35.9	6.21
4	22.269329	2 194 072.8	3.7165	28.3	4.95
5	22.460610	2 098 614.2	4.7654	23.2	4.10
7	22.962888	1 925 685.1	6.8267	17.1	3.03
10	23.995268	1 703 311.2	9.8765	12.2	2.16
20	29.340049	1 195 252.9	19.9390	6.3	1.06
30	36.545145	910 331.2	29.9614	4.3	.67
40	44.713122	741 675.5	39.9734	3.3	.46
50	53.394896	636 945.3	49.9812	2.8	.33
70	71.501327	529 576.0	69.9919	2.3	.14
90	90	$5 \cdot 10^5$	90	2.2	0

TABLE A-32.- $h_f = 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	30.139393	3 843 004.7	-1.1833	142.1	20.65
.1	30.139543	3 822 890.6	-1.0083	130.4	19.34
.2	30.139994	3 803 825.3	-.8417	120.5	18.18
.3	30.140746	3 785 646.4	-.6820	111.9	17.14
.4	30.141798	3 768 225.1	-.5283	104.3	16.20
.5	30.143150	3 751 457.8	-.3797	97.7	15.35
.7	30.146757	3 719 562.7	-.0952	86.6	13.88
1	30.154420	3 674 936.2	.3070	73.7	12.10
2	30.199454	3 543 398.8	1.5221	48.6	8.34
3	30.274362	3 426 768.0	2.6402	35.9	6.28
4	30.378921	3 318 776.8	3.7136	28.3	5.00
5	30.512822	3 216 948.1	4.7632	23.2	4.13
7	30.867030	3 027 621.9	6.8253	17.1	3.05
10	31.6064375	2 772 989.0	9.8756	12.2	2.17
20	35.644126	2 124 626.1	19.9386	6.3	1.07
30	41.500985	1 703 701.9	29.9612	4.3	.68
40	48.510701	1 429 322.9	39.9732	3.3	.47
50	56.228179	1 248 558.0	49.9811	2.8	.33
70	72.795611	1 054 833.1	69.9918	2.3	.14
90	90	10^6	90	2.2	0

TABLE A-33.- $h_f = 2 \cdot 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	40.395876	5 567 983.5	-1.2019	142.7	20.98
.1	40.395979	5 547 864.2	-1.0251	131.0	19.64
.2	40.396287	5 528 783.1	-.8569	120.9	18.45
.3	40.396799	5 510 577.6	-.6959	112.2	17.38
.4	40.397517	5 493 119.2	-.5411	104.6	16.42
.5	40.398440	5 476 304.1	-.3914	98.0	15.56
.7	40.400901	5 444 281.5	-.1052	86.7	14.05
1	40.406130	5 399 384.1	.2989	73.8	12.24
2	40.436876	5 266 256.1	1.5177	48.7	8.42
3	40.488066	5 146 986.5	2.6374	35.9	6.33
4	40.559619	5 035 326.9	3.7117	28.3	5.03
5	40.651427	4 928 825.0	4.7618	23.2	4.16
7	40.895209	4 727 270.6	6.8244	17.1	3.06
10	41.408343	4 447 723.5	9.8751	12.2	2.18
20	44.303033	3 676 763.7	19.9384	6.3	1.07
30	48.734292	3 112 210.0	29.9610	4.3	.68
40	54.309403	2 707 063.1	39.9732	3.3	.47
50	60.689820	2 421 011.3	49.9811	2.8	.33
70	74.901608	2 095 749.1	69.9918	2.3	.14
90	90	$2 \cdot 10^6$	90	2.2	0

TABLE A-34.- $h_f = 5 \cdot 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	55.890018	9 559 082.3	-1.2191	143.3	21.28
.1	55.890077	9 538 958.2	-1.0407	131.5	19.91
.2	55.890254	9 519 862.7	-.8711	121.3	18.69
.3	55.890550	9 501 633.1	-.7088	112.6	17.61
.4	55.8909635	9 484 140.8	-.5528	104.9	16.63
.5	55.891495	9 467 282.2	-.4022	98.2	15.75
.7	55.892914	9 435 143.6	-.1144	86.9	14.21
1	55.895928	9 389 999.6	.2916	73.9	12.36
2	55.913655	9 255 422.4	1.5137	48.7	8.49
3	55.943184	9 133 743.1	2.6350	35.9	6.37
4	55.984496	9 018 722.0	3.7100	28.3	5.06
5	56.037561	8 907 919.0	4.7606	23.2	4.18
7	56.178786	8 695 007.6	6.8237	17.1	3.08
10	56.477549	8 391 857.4	9.8747	12.2	2.19
20	58.199455	7 495 170.0	19.9383	6.3	1.08
30	60.944766	6 760 931.7	29.9610	4.3	.68
40	64.558780	6 177 275.3	39.9731	3.3	.47
50	68.871272	5 728 643.8	49.9811	2.8	.33
70	78.942199	5 173 719.0	69.9918	2.3	.14
90	90	$5 \cdot 10^6$	90	2.2	0

TABLE A-35 .- $h_f = 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	67.071332	15 222 522.2	-1.2281	143.6	21.43
.1	67.071369	15 202 395.8	-1.0488	131.7	20.05
.2	67.071480	15 183 292.9	-.8784	121.5	18.82
.3	67.071664	15 165 051.0	-.7154	112.7	17.72
.4	67.071923	15 147 541.4	-.5589	105.1	16.74
.5	67.072255	15 130 660.6	-.4078	98.3	15.84
.7	67.073141	15 098 462.7	-.1191	87.0	14.30
1	67.075023	15 053 192.7	.2878	74.0	12.43
2	67.086096	14 917 874.8	1.5117	48.7	8.52
3	67.104543	14 794 962.6	2.6337	35.9	6.39
4	67.130358	14 678 219.0	3.7092	28.3	5.07
5	67.163528	14 565 207.4	4.7600	23.2	4.19
7	67.251867	14 346 439.6	6.8234	17.1	3.08
10	67.439042	14 031 003.1	9.8745	12.2	2.19
20	68.525323	13 066 191.7	19.9382	6.3	1.08
30	70.282040	12 232 515.4	29.9609	4.3	.68
40	72.636113	11 534 280.5	39.9731	3.3	.47
50	75.497615	10 971 710.7	49.9810	2.8	.33
70	82.342801	10 238 920.0	69.9918	2.3	.14
90	90	10^7	90	2.2	0

TABLE A-36.- $h_f = 2 \cdot 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	76.001727	25 733 199.2	-1.2343	143.9	21.54
.1	76.001748	25 713 071.1	-1.0543	131.9	20.15
.2	76.001814	25 693 963.1	-.8834	121.7	18.91
.3	76.001922	25 675 712.8	-.7200	112.8	17.80
.4	76.002075	25 658 191.5	-.5630	105.1	16.81
.5	76.002271	25 641 295.5	-.4116	98.4	15.91
.7	76.002793	25 609 057.1	-.1223	87.0	14.35
1	76.003902	25 563 701.1	.2853	74.0	12.47
2	76.010428	25 427 877.1	1.5104	48.7	8.55
3	76.021301	25 304 122.2	2.6329	35.9	6.41
4	76.036518	25 186 200.6	3.7087	28.3	5.08
5	76.056074	25 071 677.6	4.7596	23.2	4.19
7	76.108170	24 848 896.3	6.8232	17.1	3.09
10	76.218625	24 525 012.7	9.8744	12.2	2.19
20	76.861558	23 512 689.4	19.9382	6.3	1.08
30	77.907815	22 607 568.3	29.9609	4.3	.68
40	79.321135	21 823 836.3	39.9731	3.3	.47
50	81.055039	21 172 757.5	49.9810	2.8	.33
70	85.254372	20 294 534.7	69.9918	2.3	.14
90	90	$2 \cdot 10^7$	90	2.2	0

TABLE A-37.- $h_f = 5 \cdot 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	83.50153948	56 154 315.5	-1.2391	144.0	21.63
.1	83.50154942	56 134 186.1	-1.0587	132.0	20.22
.2	83.50157924	56 115 074.2	-.8873	121.8	18.98
.3	83.50162894	56 096 817.3	-.7235	112.9	17.86
.4	83.50169852	56 079 286.8	-.5663	105.2	16.86
.5	83.50178798	56 062 379.0	-.4145	98.4	15.96
.7	83.50202655	56 030 109.0	-.1248	87.1	14.40
1	83.50253349	55 984 685.9	.2833	74.0	12.51
2	83.50551521	55 848 467.4	1.5093	48.7	8.56
3	83.51048370	55 724 055.6	2.6323	35.9	6.42
4	83.51743738	55 605 215.5	3.7083	28.3	5.09
5	83.52637405	55 489 513.6	4.7594	23.3	4.20
7	83.55018445	55 263 600.1	6.8230	17.1	3.09
10	83.60068150	54 933 114.9	9.8743	12.2	2.19
20	83.89498299	53 883 431.6	19.9382	6.3	1.08
30	84.37519577	52 921 394.4	29.9609	4.3	.68
40	85.02631236	52 068 370.7	39.9731	3.3	.47
50	85.82813460	51 344 180.5	49.9810	2.8	.33
70	87.78160340	50 342 710.6	69.9918	2.3	.14
90	90	$5 \cdot 10^7$	90	2.2	0

TABLE A-38.- $h_f = 10^8$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	86.56126936	106 325 018.1	-1.2411	144.1	21.66
.1	86.56127460	106 304 888.2	-1.0604	132.1	20.25
.2	86.56129033	106 285 774.8	-.8889	121.8	19.00
.3	86.56131655	106 267 515.3	-.7250	113.0	17.89
.4	86.56135326	106 249 981.1	-.5676	105.3	16.89
.5	86.56140045	106 233 068.6	-.4157	98.5	15.98
.7	86.56152630	106 200 786.1	-.1258	87.1	14.41
1	86.56179372	106 155 336.3	.2825	74.1	12.52
2	86.56336666	106 018 961.0	1.5089	48.8	8.57
3	86.56598767	105 894 288.0	2.6321	35.9	6.42
4	86.56965597	105 775 082.7	3.7081	28.3	5.09
5	86.57437041	105 658 912.1	4.7592	23.3	4.20
7	86.58693161	105 431 752.9	6.8229	17.1	3.09
10	86.61357279	105 098 641.1	9.8743	12.2	2.19
20	86.76887740	104 034 062.9	19.9382	6.3	1.08
30	87.02241931	103 049 234.5	29.9609	4.3	.68
40	87.36643337	102 168 301.8	39.9731	3.3	.47
50	87.79040556	101 414 469.7	49.9810	2.8	.33
70	88.82450824	100 362 576.6	69.9918	2.3	.14
90	90	10^8	90	2.2	0

TABLE A-39.- $\rho_M = 5 \cdot 10^4$ METERS, $h_i = 2000$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
-2	381.54	49 984.00	-2.07986	16.00	1.394
-1.5	819.49	49 984.65	-1.57769	15.35	1.356
-1	1257.46	49 985.28	-1.07559	14.72	1.319
-.5	1695.41	49 985.86	-.57357	14.14	1.284
a-.15	2001.96	49 986.26	-.22220	13.74	1.260
0	2133.32	49 986.42	-.07163	13.58	1.250
.5	2571.15	49 986.95	.43024	13.05	1.217
1	3008.87	49 987.45	.93205	12.55	1.186
1.5	3446.45	49 987.93	1.43379	12.07	1.156
2	3883.85	49 988.38	1.93547	11.62	1.126
3	4758.02	49 989.21	2.93865	10.79	1.071
4	5631.11	49 989.97	3.94161	10.03	1.019
5	6502.87	49 990.65	4.94437	9.35	.971
7	8241.41	49 991.82	6.94935	8.18	.884
10	10832.90	49 993.20	9.95569	6.80	.773

^a E_M undergoes a sign reversal.

TABLE A-40.- $h_f = 0$ AND $h_i = 10^6$ METERS

E_{M_1} , deg	E_{M_f} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{M_1} - E$, mrad
-30.2	-2.009072	3 542 288.36	-30.21356	48.5	0.237
-30.3	-3.273248	3 396 543.24	-30.30748	33.4	.131
-30.4	-4.173030	3 300 759.65	-30.40533	27.3	.093
-30.5	-4.912887	3 225 608.30	-30.50419	23.6	.073
-30.6	-5.557067	3 162 434.85	-30.60347	21.1	.061
-30.8	-6.666096	3 058 046.62	-30.80261	17.9	.046
-31	-7.621136	2 972 191.18	-31.00211	15.8	.037
-31.5	-9.622034	2 803 315.61	-31.50145	12.6	.025
-32	-11.297885	2 672 451.96	-32.00112	10.8	.020
-32.5	-12.777242	2 564 378.04	-32.50092	9.6	.016
-33	-14.122003	2 471 852.37	-33.00079	8.7	.014
-34	-16.536323	2 318 514.52	-34.00061	7.5	.011
-35	-18.700305	2 193 992.94	-35.00051	6.7	.009
-37	-22.559037	1 998 977.97	-37.00038	5.6	.007
-40	-27.650022	1 786 927.65	-40.00028	4.6	.005
-45	-35.150 014	1 548 038.29	-45.00020	3.7	.003
-50	-41.988 988	1 387 165.40	-50.00015	3.2	.003
-55	-48.452 358	1 271 870.73	-55.00011	2.9	.002
-60	-54.678 483	1 186 633.75	-60.00009	2.6	.002
-70	-66.703 617	1 075 474.09	-70.00005	2.3	.001
-90	-90	10^6	-90	2.2	0

APPENDIX B

This appendix contains tables of refraction corrections for

$N_0 = 0.000325$

$H_S = 6735$ meters

$R_0 = 6,378,165$ meters

$h_i = 0$ unless otherwise specified

The tables B-1 through B-40 were generated by using the set of three nonlinear differential equations to obtain the ray path. An integration step size of 5000 meters was used. The integrator was a fourth-order Runge-Kutta-Gill integrator.

The following definitions are used.

N_0 = modulus of refraction at zero altitude above R_0

H_S = atmospheric scale height

R_0 = radius of Earth, value is not critical

h_i = initial altitude of ray path above R_0

h_f = final altitude of ray path above R_0

E_{Mi} = initial measured elevation angle of ray path

E_{Mf} = final elevation angle of ray path

ρ_M = measured range = $\int n ds$

ρ = geometric, straight-line, value of range

E = geometric elevation angle

TABLE B-1.- $\rho_M = 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.0542	999.68	-0.00138	0.32	0.024
.1	1.80	999.68	.09862	.32	.024
.2	3.54	999.68	.19862	.32	.024
.3	5.29	999.68	.29862	.32	.024
.4	7.03	999.68	.39862	.32	.024
.5	8.78	999.68	.49862	.32	.024
.7	12.27	999.68	.69862	.32	.024
1	17.50	999.68	.99862	.32	.024
2	34.94	999.68	1.99862	.32	.024
3	52.37	999.68	2.99862	.32	.024
4	69.79	999.68	3.99863	.32	.024
5	87.18	999.68	4.99863	.32	.024
7	121.88	999.68	6.99864	.32	.024
10	173.65	999.68	9.99865	.32	.024
20	341.96	999.68	19.99872	.32	.022
30	499.88	999.69	29.99883	.31	.020
40	642.62	999.69	39.99897	.31	.018
50	765.83	999.69	49.99914	.31	.015
70	939.41	999.70	69.99955	.30	.008
90	999.70	999.70	90	.302	0

TABLE B-2.- $\rho_M = 2 \cdot 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.217	1999.35	-0.00276	0.65	0.048
.1	3.71	1999.35	.09724	.65	.048
.2	7.20	1999.35	.19724	.65	.048
.3	10.69	1999.35	.29724	.65	.048
.4	14.17	1999.35	.39724	.65	.048
.5	17.66	1999.35	.49724	.65	.048
.7	24.64	1999.35	.69724	.65	.048
1	35.11	1999.35	.99724	.65	.048
2	69.99	1999.35	1.99725	.65	.048
3	104.86	1999.36	2.99725	.64	.048
4	139.68	1999.36	3.99726	.64	.048
5	174.47	1999.36	4.99727	.64	.048
7	243.88	1999.36	6.99729	.64	.047
10	347.40	1999.37	9.99733	.63	.047
20	684.02	1999.38	19.99750	.62	.044
30	999.86	1999.40	29.99772	.60	.040
40	1285.33	1999.41	39.99801	.59	.035
50	1531.74	1999.42	49.99835	.58	.029
70	1878.88	1999.43	69.99914	.57	.015
90	1999.44	1999.44	90	.562	0

TABLE B-3.- $\rho_M = 5 \cdot 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1.356	4998.38	-0.00691	1.62	0.121
.1	10.08	4998.38	.09310	1.62	.121
.2	18.80	4998.38	.19310	1.62	.120
.3	27.53	3998.38	.29310	1.62	.120
.4	36.25	4998.38	.39310	1.62	.120
.5	44.98	4998.38	.49311	1.62	.120
.7	62.42	4998.38	.69311	1.62	.120
1	88.59	4998.39	.99312	1.61	.120
2	175.80	4998.40	1.99316	1.60	.119
3	262.96	4998.41	2.99319	1.59	.119
4	350.03	4998.42	3.99323	1.58	.118
5	437.00	4998.43	4.99326	1.57	.118
7	610.51	4998.45	6.99335	1.55	.116
10	869.32	4998.48	9.99348	1.52	.114
20	1710.85	4998.57	19.99402	1.43	.104
30	2500.39	4998.64	29.99469	1.36	.093
40	3213.95	4998.71	39.99546	1.29	.079
50	3829.87	4998.76	49.99629	1.24	.065
70	4697.54	4998.83	69.99810	1.17	.033
90	4998.85	4998.85	90	1.147	0

TABLE B-4.- $\rho_M = 10^4$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	5.424	9996.75	-0.01381	3.25	0.241
.1	22.87	9996.76	.08620	3.24	.241
.2	40.32	9996.76	.18621	3.24	.241
.3	57.77	9996.76	.28622	3.24	.240
.4	75.22	9996.77	.38623	3.23	.240
.5	92.67	9996.77	.48625	3.23	.240
.7	127.57	9996.78	.68627	3.22	.240
1	179.91	9996.79	.98631	3.21	.239
2	354.34	9996.83	1.98643	3.17	.237
3	528.67	9996.87	2.98656	3.13	.235
4	702.83	9996.91	3.98668	3.09	.232
5	876.77	9996.95	4.98681	3.05	.230
7	1223.81	9997.03	6.98708	2.97	.225
10	1741.43	9997.14	9.98749	2.86	.218
20	3424.44	9997.45	19.98896	2.55	.193
30	5003.29	9997.71	29.99052	2.29	.165
40	6430.07	9997.91	39.99211	2.09	.138
50	7661.49	9998.06	49.99371	1.94	.110
70	9396.01	9998.25	69.99688	1.75	.055
90	9998.31	9998.31	90	1.693	0

TABLE B-5.- $\rho_M = 2 \cdot 10^4$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	21.70	19 993.51	-0.02762	6.49	0.482
.1	56.61	19 993.53	.07243	6.47	.481
.2	91.52	19 993.54	.17248	6.46	.480
.3	126.43	19 993.56	.27253	6.44	.479
.4	161.35	19 993.58	.37257	6.42	.479
.5	196.26	19 993.59	.47262	6.41	.478
.7	266.07	19 993.62	.67272	6.38	.476
1	370.79	19 993.67	.97286	6.33	.474
2	719.77	19 993.83	1.97333	6.17	.465
3	1068.52	19 993.99	2.97380	6.01	.457
4	1416.94	19 994.14	2.97426	5.86	.449
5	1764.92	19 994.28	4.97472	5.72	.441
7	2459.14	19 994.55	6.97562	5.45	.426
10	3494.54	19 994.93	9.97693	5.07	.403
20	6860.43	19 995.92	19.98099	4.08	.332
30	10 017.12	19 996.62	29.98456	3.38	.269
40	12 868.97	19 997.10	39.98772	2.90	.214
50	15 329.73	19 997.44	49.99054	2.56	.165
70	18 794.92	19 997.81	69.9950	2.19	.078
90	19 997.92	19 997.92	90	2.077	0

TABLE B-6.- $\rho_M = 5 \cdot 10^4$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	135.19	49 983.85	-0.06884	16.15	1.202
.1	223.29	49 983.96	.03145	16.04	1.196
.2	310.78	49 984.06	.13175	15.94	1.191
.3	398.27	49 984.16	.23204	15.84	1.186
.4	485.76	49 984.26	.33233	15.74	1.181
.5	573.24	49 984.36	.43262	15.64	1.176
.7	748.20	49 984.56	.63319	15.44	1.166
1	1010.60	49 984.85	.93404	15.15	1.151
2	1884.63	49 985.77	1.93676	14.23	1.104
3	2758.52	49 986.61	2.93933	13.39	1.059
4	3631.11	49 987.39	3.94177	12.61	1.016
5	4502.44	49 988.10	4.94407	11.90	.976
7	6240.25	49 989.36	6.94831	10.64	.902
10	8831.00	49 990.91	9.95389	9.09	.805
20	17 244.95	49 994.12	19.96748	5.88	.568
30	25 126.39	49 995.74	29.97627	4.26	.414
40	32 239.95	49 996.63	39.98244	3.37	.306
50	38 373.35	49 997.15	49.98713	2.85	.225
70	47 003.48	49 997.67	69.99420	2.33	.101
90	49 997.81	49 997.81	90	2.188	0

TABLE B-7.- $\rho_M = 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	545.55	99 968.27	-0.13622	31.73	2.379
.1	722.04	99 968.68	-.03516	31.32	2.359
.2	898.51	99 969.08	.06599	30.92	2.339
.3	1074.94	99 969.47	.16712	30.53	2.319
.4	1251.35	99 969.85	.26824	30.15	2.230
.5	1427.72	99 970.23	.36934	29.77	2.280
.7	1780.39	99 970.97	.57151	29.03	2.243
1	2309.16	99 972.03	.87466	27.97	2.188
2	4069.69	99 975.20	1.88438	24.80	2.018
3	5826.90	99 977.88	2.89300	22.12	1.867
4	7580.47	99 980.16	3.90068	19.84	1.733
5	9330.09	99 982.11	4.90754	17.89	1.614
7	12 815.99	99 985.22	6.91920	14.78	1.410
10	18 005.61	99 988.50	9.93278	11.50	1.173
20	34 821.43	99 993.67	19.95903	6.33	.715
30	50 539.36	99 995.64	29.97216	4.36	.486
40	64 705.72	99 996.60	39.98017	3.40	.346
50	76 906.51	99 997.14	49.98577	2.86	.248
70	94 053.74	99 997.67	69.99371	2.33	.110
90	99 997.81	99 997.81	90	2.189	0

TABLE B-8.- $\rho_M = 2 \cdot 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	2218.27	199 940.80	-0.26226	59.20	4.577
.1	2582.28	199 942.28	-.15792	57.72	4.502
.2	2945.90	199 943.70	-.05369	56.30	4.428
.3	3309.13	199 945.07	.05044	54.93	4.356
.4	3671.98	199 946.40	.15446	53.60	4.285
.5	4034.47	199 947.68	.25838	52.32	4.217
.7	4758.39	199 950.11	.46594	49.89	4.085
1	5841.74	199 953.47	.77661	46.53	3.899
2	9433.42	199 962.55	1.80714	37.45	3.366
3	12 999.58	199 969.19	2.83145	30.81	2.942
4	16 544.43	199 974.15	3.85109	25.85	2.599
5	20 070.73	199 977.92	4.86716	22.08	2.318
7	27 074.00	199 983.15	6.89162	16.85	1.892
10	37 464.34	199 987.75	9.91609	12.25	1.464
20	70 991.87	199 993.64	19.95416	6.36	.800
30	102 223.55	199 995.63	29.97002	4.37	.523
40	130 303.77	199 996.60	39.97901	3.40	.366
50	154 439.07	199 997.14	49.98508	2.86	.260
70	188 285.05	199 997.67	69.99347	2.33	.114
90	199 997.81	199 997.81	90	2.189	0

TABLE B-9.- $\rho_M = 5 \cdot 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	15 012 .93	499 900 .62	-0 .52262	99 .38	9 .121
.1	16 039 .81	499 905 .54	-.40467	94 .46	8 .808
.2	17 057 .72	499 910 .09	-.28774	89 .91	8 .513
.3	18 067 .34	499 914 .28	-.17174	85 .72	8 .233
.4	19 069 .26	499 918 .16	-.05661	81 .84	7 .969
.5	20 064 .02	499 921 .76	.05772	78 .24	7 .719
.7	22 034 .03	499 928 .22	.28418	71 .78	7 .257
1	24 946 .43	499 936 .37	.61913	63 .63	6 .648
2	34 382 .39	499 954 .71	1 .70559	45 .29	5 .138
3	43 543 .38	499 965 .42	2 .76253	34 .58	4 .145
4	52 534 .49	499 972 .26	3 .80227	27 .74	3 .451
5	61 409 .90	499 976 .95	4 .83131	23 .05	2 .944
7	78 919 .24	499 982 .88	6 .87049	17 .12	2 .260
10	104 741 .06	499 987 .70	9 .90483	12 .30	1 .661
20	187 450 .73	499 993 .64	19 .95120	6 .36	.852
30	263 899 .71	499 995 .63	29 .96873	4 .37	.546
40	332 193 .71	499 996 .60	39 .97832	3 .40	.378
50	390 573 .37	499 997 .14	49 .98467	2 .86	.267
70	471 960 .47	499 997 .67	69 .99333	2 .33	.116
90	499 997 .81	499 997 .81	90	2 .189	0

TABLE B-10 - $\rho_M = 10^6$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	66 235.36	999 893.94	-0.67598	106.06	11.798
.1	68 478.30	999 899.91	-.54614	100.09	11.277
.2	70 686.96	999 905.32	-.41853	94.68	10.795
.3	72 850.36	999 910.23	-.29292	89.77	10.348
.4	74 986.97	999 914.71	-.16911	85.29	9.933
.5	77 094.84	999 918.81	-.04693	81.19	9.546
.7	81 234.60	999 926.04	.19316	73.96	8.846
1	87 285.52	999 934.96	.54438	65.04	7.952
2	106 521.49	999 954.33	1.66336	45.67	5.875
3	124 899.08	999 965.29	2.73615	34.71	4.605
4	142 792.44	999 972.21	3.78453	27.79	3.761
5	160 373.72	999 976.93	4.81870	23.07	3.164
7	194 910.58	999 982.87	6.86330	17.13	2.386
10	245 606.77	999 987.70	9.90105	12.30	1.727
20	406 630.70	999 993.64	19.95021	6.36	.869
30	553 862.37	999 995.63	29.96830	4.37	.553
40	684 187.84	999 996.60	39.97809	3.40	.382
50	794 746.38	999 997.14	49.98454	2.86	.270
70	947 643.72	999 997.67	69.99328	2.33	.117
90	999 997.81	999 997.81	90	2.189	0

TABLE B-11.- $\rho_M = 2 \cdot 10^6$ METERS

E_{Mi} , deg	hf, meters	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	281 003.8	1 999 892.0	-0.7547	108.0	13.17
.1	285 555.9	1 999 898.3	-.6185	101.7	12.54
.2	290 007.6	1 999 904.0	-.4853	96.0	11.96
.3	294 369.9	1 999 909.1	-.3546	90.9	11.43
.4	298 652.2	1 999 913.7	-.2263	86.3	10.93
.5	302 862.6	1 999 918.0	-.1000	82.0	10.47
.7	311 095.2	1 999 925.4	.1471	74.6	9.65
1	323 054.6	1 999 934.5	.5067	65.5	8.61
2	360 672.0	1 999 954.2	1.6422	45.8	6.24
3	396 260.9	1 999 965.2	2.7229	34.8	4.84
4	430 716.3	1 999 972.2	3.7757	27.8	3.92
5	464 438.0	1 999 976.9	4.8124	23.1	3.27
7	530 387.2	1 999 982.9	6.8597	17.1	2.45
10	626 613.0	1 999 987.7	9.8992	12.3	1.76
20	928 389.5	1 999 993.6	19.9497	6.4	.88
30	1 119 761.0	1 999 995.6	29.9681	4.4	.56
40	1 436 734.5	1 999 996.6	39.9780	3.4	.38
50	1 635 594.1	1 999 997.1	49.9845	2.9	.27
70	1 907 604.9	1 999 997.7	69.9933	2.3	.12
90	1 999 997.8	1 999 997.8	90	2.189	0

TABLE B-12.- $\rho_M = 5 \cdot 10^6$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1 670 886.4	4 999 890.9	-0.8019	109.1	14.00
.1	1 680 561.7	4 999 897.4	-.6620	102.6	13.30
.2	1 689 996.1	4 999 903.1	-.5253	96.9	12.66
.3	1 699 216.4	4 999 908.4	-.3917	91.6	12.07
.4	1 708 824.6	4 999 913.1	-.2606	86.9	11.53
.5	1 717 104.1	4 999 917.4	-.1319	82.6	11.03
.7	1 734 373.2	4 999 925.0	.1195	75.0	10.13
1	1 759 354.8	4 999 934.3	.4841	65.7	9.00
2	1 837 321.7	4 999 954.1	1.6295	45.9	6.47
3	1 910 493.3	4 999 965.2	2.7150	34.8	4.97
4	1 980 949.9	4 999 972.2	3.7703	27.8	4.01
5	2 049 610.4	4 999 976.9	4.8086	23.1	3.34
7	2 183 167.1	4 999 982.9	6.8576	17.1	2.49
10	2 376 525.4	4 999 987.7	9.8980	12.3	1.78
20	2 972 909.5	4 999 993.6	19.9494	6.4	.88
30	3 498 111.7	4 999 995.6	29.9680	4.4	.56
40	3 949 474.5	4 999 996.6	39.9779	3.4	.39
50	4 323 674.4	4 999 997.1	49.9844	2.9	.27
70	4 829 578.3	4 999 997.7	69.9932	2.3	.12
90	4 999 997.8	4 999 997.8	90	2.189	0

TABLE B-13.- $\rho_M = 10^7$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	5 405 654.1	9 999 890.5	-0.8177	109.5	14.27
.1	5 418 991.8	9 999 897.0	-.6765	103.0	13.55
.2	5 431 988.2	9 999 902.9	-.5387	97.1	12.89
.3	5 444 681.8	9 999 908.1	-.4040	91.9	12.29
.4	5 457 105.5	9 999 912.9	-.2720	87.1	11.73
.5	5 469 287.7	9 999 917.3	-.1425	82.7	11.21
.7	5 893 021.5	9 999 924.9	.1103	75.1	10.29
1	5 527 325.3	9 999 934.2	.4766	65.8	9.14
2	5 634 247.4	9 999 954.1	1.6253	45.9	6.54
3	5 734 509.0	9 999 965.2	2.7124	34.8	5.02
4	5 831 040.4	9 999 972.2	3.7686	27.8	4.04
5	5 925 128.6	9 999 976.9	4.8074	23.1	3.36
7	6 108 238.1	9 999 982.9	6.8568	17.1	2.50
10	6 373 596.4	9 999 987.7	9.8977	12.3	1.79
20	7 194 013.2	9 999 993.6	19.9493	6.4	.88
30	7 918 710.1	9 999 995.6	29.9679	4.4	.56
40	8 542 936.6	9 999 996.6	39.9779	3.4	.39
50	9 061 321.8	9 999 997.1	49.9844	2.9	.27
70	9 763 274.0	9 999 997.7	69.9932	2.3	.12
90	9 999 997.8	9 999 997.8	90	2.189	0

TABLE B-14.- $\rho_M = 2 \cdot 10^7$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	14 526 397.3	19 999 890.4	-0.8256	109.6	14.41
.1	14 541 504.6	19 999 896.9	-.6837	103.1	13.68
.2	14 556 225.5	19 999 902.7	-.5454	97.3	13.01
.3	14 570 604.3	19 999 908.0	-.4102	92.0	12.39
.4	14 584 678.4	19 999 912.8	-.2777	87.2	11.83
.5	14 598 480.1	19 999 917.2	-.1478	82.8	11.31
.7	14 625 374.0	19 999 924.8	.1057	75.2	10.37
1	14 664 259.8	19 999 934.1	.4728	65.9	9.20
2	14 785 617.9	19 999 954.1	1.6231	45.9	6.58
3	14 899 685.1	19 999 965.2	2.7111	34.8	5.04
4	15 009 786.9	19 999 972.2	3.7677	27.8	4.05
5	15 117 376.5	19 999 976.9	4.8067	23.1	3.37
7	15 327 548.0	19 999 982.9	6.8565	17.1	2.50
10	15 633 946.4	19 999 987.7	9.8975	12.3	1.79
20	16 594 141.7	19 999 993.6	19.9493	6.4	.89
30	17 457 107.9	19 999 995.6	29.9679	4.4	.56
40	18 210 400.4	19 999 996.6	39.9779	3.4	.39
50	18 842 352.9	19 999 997.1	49.9844	2.9	.27
70	19 706 526.7	19 999 997.7	69.9932	2.3	.12
90	19 999 997.8	19 999 997.8	90	2.189	0

TABLE B-15.- $\rho_M = 5 \cdot 10^7$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	43 935 130.6	49 999 890.2	-0.8303	109.8	14.49
.1	43 950 868.1	49 999 896.8	-.6880	103.2	13.75
.2	43 966 205.8	49 999 902.7	-.5494	97.3	13.08
.3	43 981 189.9	49 999 908.0	-.4139	92.0	12.46
.4	43 995 859.4	49 999 912.8	-.2812	87.2	11.89
.5	44 010 247.9	49 999 917.1	-.1510	82.9	11.36
.7	44 038 294.6	49 999 924.8	.1029	75.2	10.42
1	44 078 870.9	49 999 934.1	.4705	65.9	9.24
2	44 205 712.7	49 999 954.1	1.6219	45.9	6.60
3	44 325 257.0	49 999 965.2	2.7103	34.8	5.06
4	44 440 963.8	49 999 972.2	3.7671	27.8	4.06
5	44 554 340.5	49 999 976.9	4.8063	23.1	3.38
7	44 776 708.8	49 999 982.9	6.8563	17.1	2.51
10	45 102 996.0	49 999 987.7	9.8974	12.3	1.79
20	46 141 311.7	49 999 993.6	19.9492	6.4	.89
30	47 094 061.6	49 999 995.6	29.9679	4.4	.56
40	47 940 085.7	49 999 996.6	39.9779	3.4	.39
50	48 659 684.2	49 999 997.1	49.9844	2.9	.27
70	49 657 595.1	49 999 997.7	69.9932	2.3	.12
90	49 999 997.8	49 999 997.8	90	2.189	0

TABLE B-16 .- $\rho_M = 10^8$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	93 732 470.6	99 999 890.2	-0.8319	109.8	14.52
.1	93 748 304.5	99 999 896.7	-.6895	103.3	13.78
.2	93 763 737.4	99 999 902.6	-.5507	97.4	13.10
.3	93 778 815.6	99 999 907.9	-.4151	92.1	12.48
.4	93 793 578.6	99 999 912.7	-.2823	87.3	11.91
.5	93 808 060.0	99 999 917.1	-.1520	82.9	11.38
.7	93 836 291.6	99 999 924.8	.1020	75.2	10.44
1	93 877 145.0	99 999 934.1	.4698	65.9	9.25
2	94 004 936.0	99 999 954.0	1.6215	46.0	6.61
3	94 125 502.4	99 999 965.2	2.7100	34.8	5.06
4	94 242 324.0	99 999 972.2	3.7670	27.8	4.07
5	94 356 915.7	99 999 976.9	4.8062	23.1	3.38
7	94 582 023.5	99 999 982.9	6.8562	17.1	2.51
10	94 913 187.5	99 999 987.7	9.8973	12.3	1.79
20	95 973 726.5	99 999 993.6	19.9492	6.4	.89
30	96 955 648.4	99 999 995.6	29.9679	4.4	.56
40	97 834 424.1	99 999 996.6	39.9779	3.4	.39
50	98 586 823.0	99 999 997.1	49.9844	2.9	.27
70	99 637 550.1	99 999 997.7	69.9932	2.3	.12
90	99 999 997.8	99 999 997.8	90	2.189	0

TABLE B-17 .- $h_f = 10$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.084433	13 573.32	-0.01875	4.41	0.327
.1	.130878	4 963.38	.09314	1.61	.120
.2	.217092	2 747.41	.19621	.89	.066
.3	.311655	1 873.48	.29741	.61	.045
.4	.408814	1 416.80	.39804	.46	.034
.5	.507079	1 137.88	.49843	.37	.027
.7	.705074	815.58	.69887	.26	.020
1	1.003558	571.97	.99921	.19	.014
2	2.001781	286.41	1.99960	.09	.007
3	3.001187	191.04	2.99974	.06	.005
4	4.000890	143.34	3.99980	.05	.003
5	5.000711	114.73	4.99984	.04	.003
7	7.000507	82.05	6.99989	.03	.002
10	10.000353	57.59	9.99992	.02	.001
20	20.000171	29.24	19.99996	.01	.001
30	30.000108	20.00	29.99998	.01	0
40	40.000074	15.56	39.99998	.01	0
50	50.000052	13.05	49.99999	0	0
70	70.000023	10.64	69.99999	0	0
90	90	10	90	0	0

TABLE B-18 .- $h_f = 20$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.119426	19 194.52	-0.02651	6.23	0.463
.1	.155765	8 961.13	.08763	2.91	.216
.2	.232943	5 293.71	.19269	1.72	.128
.3	.322897	3 679.36	.29492	1.19	.089
.4	.417448	2 803.68	.39613	.91	.068
.5	.514064	2 260.08	.49688	.73	.054
.7	.710114	1 625.32	.69776	.53	.039
1	1.007105	1 141.92	.99842	.37	.028
2	2.003561	572.56	1.99921	.19	.014
3	3.002374	382.00	2.99947	.12	.009
4	4.001780	286.65	3.99961	.09	.007
5	5.001422	229.44	4.99968	.07	.006
7	7.001014	164.10	6.99978	.05	.004
10	10.000706	115.17	9.99984	.04	.003
20	20.000342	58.48	19.99992	.02	.001
30	30.000216	40.00	29.99995	.01	.001
40	40.000148	31.11	39.99997	.01	.001
50	50.000104	26.11	49.99998	.01	0
70	70.000045	21.28	69.99999	.01	0
90	90	20	90	.01	0

TABLE B-19 .- $h_f = 50$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.188922	30 344.24	-0.04188	9.84	0.731
.1	.213756	18 264.89	.07481	5.92	.440
.2	.275121	12 060.27	.18337	3.91	.290
.3	.354530	8 754.18	.28793	2.83	.211
.4	.442370	6 801.98	.39062	2.21	.164
.5	.534500	5 538.67	.49236	1.79	.133
.7	.725045	4 020.77	.69446	1.30	.097
1	1.017688	2 839.84	.99609	.92	.068
2	2.008899	1 429.51	1.99803	.46	.034
3	3.005937	954.42	2.99869	.31	.023
4	4.004452	716.38	3.99901	.23	.017
5	5.003559	573.48	4.99921	.19	.014
7	7.002536	410.20	6.99944	.13	.010
10	10.001766	287.91	9.99961	.09	.007
20	20.000856	146.19	19.99981	.05	.003
30	30.000539	100.00	29.99988	.03	.002
40	40.000371	77.79	39.99992	.03	.001
50	50.000261	65.27	49.99994	.02	.001
70	70.000113	53.21	69.99997	.02	0
90	90	50	90	.02	0

TABLE B-20.- $h_f = 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.267394	42 901.60	-0.05914	13.88	1.032
.1	.285481	29 742.48	.05905	9.61	.715
.2	.333915	21 468.43	.17046	6.93	.516
.3	.401869	16 329.30	.27754	5.27	.392
.4	.481143	13 006.31	.38211	4.20	.312
.5	.567008	10 740.41	.48523	3.47	.258
.7	.749330	7 907.02	.68913	2.55	.190
1	1.035129	5 631.08	.99226	1.81	.135
2	2.017788	2 852.70	1.99608	.92	.068
3	3.011882	1 906.96	2.99738	.62	.046
4	4.008913	1 431.97	3.99804	.46	.034
5	5.007127	1 146.56	4.99843	.37	.027
7	7.005080	820.26	6.99888	.26	.020
10	10.003538	575.78	9.99922	.19	.014
20	20.001714	292.37	19.99962	.09	.007
30	30.001081	200.00	29.99976	.06	.004
40	40.000744	155.57	39.99984	.05	.003
50	50.000487	131.11	49.99988	.04	.002
70	70.000227	106.42	69.99995	.03	.001
90	90	100	90	.03	0

TABLE B-21.- $h_f = 2 \cdot 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.378761	60 639.31	-0.08339	19.54	1.455
.1	.391739	46 666.26	.03596	15.00	1.118
.2	.428321	36 504.11	.14996	11.72	.873
.3	.483175	29 278.23	.25989	9.39	.700
.4	.550870	24 110.95	.36698	7.73	.576
.5	.627261	20 336.22	.47216	6.52	.486
.7	.795897	15 323.32	.67903	4.91	.366
1	1.069320	11 076.68	.98484	3.55	.265
2	2.035535	5680.41	1.99223	1.82	.136
3	3.023794	3806.42	2.99480	1.22	.091
4	4.017863	2860.75	3.99609	.92	.068
5	5.014289	2291.48	4.99688	.73	.055
7	7.010189	1639.92	6.99777	.53	.039
10	10.007097	1151.35	9.99845	.37	.027
20	20.003439	584.71	19.99925	.19	.013
30	30.002168	399.99	29.99953	.13	.008
40	40.001492	311.14	39.99967	.10	.006
50	50.001050	261.08	49.99977	.08	.004
70	70.000456	212.84	69.99990	.07	.002
90	90	200	90	.06	0

TABLE B-22.- $h_f = 5 \cdot 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.601697	95 726.69	-0.13068	30.44	2.281
.1	.609950	81 008.98	-.01020	25.65	1.923
.2	.634065	68 882.42	.10652	21.75	1.631
.3	.672337	59 044.45	.22001	18.60	1.396
.4	.722519	51 119.44	.33083	16.08	1.207
.5	.782324	44 733.39	.43952	14.06	1.056
.7	.923050	35 327.71	.65229	11.09	.833
1	1.167049	26 451.71	.96430	8.30	.623
2	2.088514	14 018.39	1.98110	4.39	.330
3	3.059691	9460.12	2.98726	2.96	.222
4	4.044929	7128.03	3.99041	2.23	.167
5	5.035983	5716.45	4.99232	1.79	.134
7	7.025684	4095.32	6.99452	1.28	.096
10	10.017902	2876.85	9.99618	.90	.067
20	20.008678	1461.60	19.99815	.46	.032
30	30.005472	999.92	29.99883	.31	.020
40	40.003765	777.83	39.99920	.24	.014
50	50.002651	652.69	49.99943	.20	.010
70	70.001150	532.09	69.99975	.17	.004
90	90	500	90	.16	0

TABLE B-23.- $h_f = 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.857288	135 030.85	-0.18215	42.03	3.179
.1	.863100	119 937.31	-.06095	37.10	2.809
.2	.880306	106 746.99	.05733	32.86	2.490
.3	.908260	95 320.38	.17300	29.23	2.217
.4	.946008	85 481.31	.28638	26.13	1.983
.5	.992433	77 036.29	.39780	23.50	1.784
.7	1.106755	63 568.37	.61589	19.33	1.468
1	1.317144	49 524.18	.93462	15.02	1.141
2	2.175924	27 458.49	1.96385	8.30	.631
3	3.119979	18 736.70	2.97536	5.66	.430
4	4.090690	14 176.72	3.98138	4.28	.325
5	5.072778	11 391.83	4.98506	3.44	.261
7	7.052041	8175.57	6.98932	2.47	.187
10	10.036307	5748.54	9.99255	1.74	.130
20	20.017613	2922.58	19.99639	.88	.063
30	30.011107	1999.67	29.99772	.60	.040
40	40.007643	1555.60	39.99843	.47	.027
50	50.005381	1305.36	49.99890	.39	.019
70	70.002334	1064.17	69.99952	.32	.008
90	90	1·10 ³	90	.30	0

TABLE B-24 .- $h_f = 2 \cdot 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1.228928	190 040.64	-0.25050	56.76	4.372
.1	1.232990	174 687.33	-.12853	51.69	3.989
.2	1.245094	160 739.98	-.00896	47.19	3.647
.3	1.265010	148 121.06	.10846	43.21	3.343
.4	1.292378	136 742.31	.22396	39.67	3.073
.5	1.326735	126 507.72	.33773	36.54	2.832
.7	1.414281	109 070.91	.56089	31.29	2.428
1	1.584333	89 051.92	.88705	25.38	1.971
2	2.347265	52 807.04	1.93353	14.92	1.160
3	3.241741	36 761.39	2.95385	10.36	.805
4	4.184234	28 038.07	3.96487	7.89	.613
5	5.148440	22 618.53	4.97171	6.37	.494
7	7.106528	16 289.89	6.97971	4.58	.354
10	10.074468	11 476.00	9.98582	3.23	.248
20	20.036178	5842.63	19.99311	1.64	.120
30	30.022819	3998.65	29.99565	1.12	.076
40	40.015704	3110.95	39.99701	.87	.052
50	50.011058	2610.61	49.99789	.73	.037
70	70.004797	2128.32	69.99909	.60	.016
90	90	$2 \cdot 10^3$	90	.56	0

TABLE B-25.- $h_f = 5 \cdot 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	2.006538	296 737.80	-0.36701	79.07	6.406
.1	2.009028	281 167.34	-.24359	73.79	5.997
.2	2.016477	266 576.73	-.12227	69.02	5.625
.3	2.208832	252 912.98	-.00281	64.70	5.285
.4	2.046003	240 125.73	.11497	60.78	4.975
.5	2.067872	228 166.43	.23124	57.20	4.691
.7	2.125087	206 543.94	.45986	50.96	4.191
1	2.241827	179 176.79	.79473	43.42	3.583
2	2.832765	119 299.04	1.86678	28.04	2.325
3	3.608672	87 019.16	2.90382	20.22	1.679
4	4.474333	67 801.43	3.92546	15.68	1.301
5	5.386647	55 308.20	4.93941	12.75	1.058
7	7.280532	40 252.29	6.95615	9.26	.765
10	10.197315	28 524.76	9.96921	6.55	.537
20	20.096301	14 586.49	19.98499	3.35	.262
30	30.060794	9991.13	29.99052	2.29	.165
40	40.041850	7775.38	39.99348	1.78	.114
50	50.029472	6525.68	49.99541	1.50	.080
70	70.102786	5320.68	69.99801	1.22	.035
90	90	$5 \cdot 10^3$	90	1.15	0

TABLE B-26.- $h_f = 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	2.938021	412 958.89	-0.466630	93.66	8.138
.1	2.939721	397 289.77	-.341115	88.18	7.700
.2	2.944815	382 404.33	-.218111	83.21	7.297
.3	2.953285	368 254.27	-.09694	78.69	6.928
.4	2.965102	354 796.77	.02257	74.57	6.587
.5	2.980227	341 993.38	.14058	70.79	6.273
.7	3.020189	318 211.74	.37270	64.11	5.712
1	3.103400	286 650.64	.71288	55.89	5.011
2	3.553655	209 203.16	1.80156	38.18	3.463
3	4.198104	160 546.09	2.85168	28.41	2.589
4	4.961647	128 634.54	3.88285	22.40	2.045
5	5.797417	106 645.75	4.90372	18.41	1.680
7	7.588740	78 904.77	6.92955	13.50	1.230
10	10.418451	56 471.61	9.95019	9.61	.869
20	20.205901	29 100.92	19.97559	4.93	.426
30	30.130187	19 962.51	29.98458	3.38	.269
40	40.089669	15 543.40	39.98938	2.63	.185
50	50.063164	13 048.31	49.99252	2.21	.131
70	70.027408	10 640.89	69.99675	1.80	.057
90	90	10^4	90	1.69	0

TABLE B-27.- $h_f = 2 \cdot 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	4.302811	571 585.28	-0.55937	101.99	9.763
.1	4.303971	555 853.61	-.43188	96.29	9.283
.2	4.307448	540 780.57	-.30662	91.13	8.842
.3	4.313237	526 319.24	-.18335	86.44	8.436
.4	4.321329	512 429.09	-.06186	82.15	8.061
.5	4.331710	499 074.83	.05806	78.22	7.713
.7	4.359273	473 853.18	.29375	71.28	7.090
1	4.417273	439 364.54	.63879	62.70	6.304
2	4.744118	347 597.78	1.74085	43.98	4.523
3	5.243782	282 166.51	2.80125	33.35	3.469
4	5.872315	234 640.57	3.84034	26.63	2.787
5	6.592962	199 362.15	4.86733	22.07	2.316
7	8.211086	151 686.89	6.90169	16.34	1.716
10	10.877763	110 601.00	9.92992	11.71	1.223
20	20.439074	57 895.53	19.96541	6.04	.604
30	30.278538	39 840.21	29.97812	4.14	.382
40	40.192074	31 055.38	39.98492	3.23	.263
50	50.135372	26 083.51	49.98938	2.71	.185
70	50.058766	21 279.77	69.99539	2.21	.080
90	90	$2 \cdot 10^4$	90	2.08	0

TABLE B-28.- $h_f = 5 \cdot 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	7.001007	876 276.84	-0.65377	105.52	11.410
.1	7.001717	860 495.16	-.52269	99.56	10.868
.2	7.003849	845 271.69	-.39411	94.18	10.369
.3	7.007400	830 559.99	-.26776	89.29	9.909
.4	7.012368	816 320.36	-.14340	84.83	9.484
.5	7.018750	802 518.61	-.02082	80.76	9.090
.7	7.035741	776 113.90	.21971	73.58	8.383
1	7.071712	739 159.96	.57096	64.73	7.488
2	7.279709	634 096.66	1.68778	45.51	5.449
3	7.613754	550 150.37	2.75773	34.62	4.228
4	8.058186	482 066.14	3.80359	27.73	3.428
5	8.595899	426 388.53	4.83565	23.03	2.868
7	9.887858	342 437.16	6.87710	17.11	2.145
10	12.186669	260 669.47	9.91175	12.29	1.540
20	21.142494	142 414.10	19.95614	6.36	.765
30	30.731851	98 940.30	29.97221	4.36	.485
40	40.506452	77 391.47	39.98084	3.40	.334
50	50.357526	65 105.31	49.98650	2.85	.236
70	70.155410	53 183.47	69.99414	2.33	.102
90	90	$5 \cdot 10^4$	90	2.19	0

TABLE B-29.- $h_f = 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	9.974972	1 214 911.35	-0.70384	106.73	12.284
.1	9.975468	1 199 105.95	-.57020	100.62	11.697
.2	9.976956	1 183 811.01	-.43927	95.11	11.157
.3	9.979436	1 168 980.19	-.31077	90.11	10.660
.4	9.982907	1 154 573.95	-.18443	85.56	10.200
.5	9.987369	1 140 558.33	-.06002	81.40	9.774
.7	9.999255	1 113 585.31	.18377	74.09	9.010
1	10.024467	1 075 433.52	.53909	65.10	8.044
2	10.171507	963 584.87	1.66496	45.66	5.848
3	10.411961	869 226.58	2.74012	34.70	4.536
4	10.739555	788 155.88	3.78930	27.78	3.677
5	11.146607	717 973.02	4.82365	23.06	3.078
7	12.165669	603 800.00	6.86802	17.12	2.303
10	14.088494	480 622.65	9.90513	12.30	1.656
20	22.257978	277 634.04	19.95278	6.36	.824
30	31.467748	195 738.05	29.97006	4.37	.522
40	41.021395	153 966.97	39.97936	3.40	.360
50	50.722945	129 865.86	49.98545	2.86	.254
70	70.314925	106 313.38	69.99369	2.33	.110
90	90	10^5	90	2.19	0

TABLE B-30.- $h_f = 2 \cdot 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	14.090710	1 694 281.1	-0.7405	107.6	12.92
.1	14.091057	1 678 459.3	-.6047	101.4	12.30
.2	14.092100	1 663 114.9	-.4718	95.8	11.72
.3	14.093838	1 648 201.5	-.3415	90.7	11.20
.4	14.096271	1 633 679.7	-.2135	86.1	10.71
.5	14.099399	1 619 515.6	-.0875	81.8	10.25
.7	14.107735	1 592 147.4	.1589	74.4	9.44
1	14.125433	1 553 159.1	.5175	65.3	8.42
2	14.229094	1 436 481.9	1.6505	45.8	6.10
3	14.400205	1 344 409.5	2.7295	34.7	4.72
4	14.636397	1 243 180.0	3.7811	27.8	3.82
5	14.934584	1 160 912.1	4.8170	23.1	3.19
7	15.702078	1 018 881.4	6.8632	17.1	2.39
10	17.219751	851 088.7	9.9017	12.3	1.72
20	24.297855	530 635.8	19.9511	6.4	.85
30	32.863259	383 564.6	29.9690	4.4	.54
40	42.012800	304 826.5	39.9786	3.4	.37
50	51.431700	258 399.0	49.9849	2.9	.26
70	70.626231	212 417.2	69.9935	2.3	.11
90	90	$2 \cdot 10^5$	90	2.2	0

TABLE B-31.- $h_f = 5 \cdot 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	21.935102	2 662 155.9	-0.7743	108.4	13.51
.1	21.935319	2 646 319.6	-.6362	102.1	12.85
.2	21.935969	2 630 931.6	-.5013	96.4	12.24
.3	21.937053	2 615 945.4	-.3692	91.2	11.68
.4	21.93857	2 601 321.6	-.2395	86.5	11.16
.5	21.940519	2 587 026.5	-.1121	82.2	10.68
.7	21.945718	2 559 309.0	.1371	74.7	9.83
1	21.956762	2 519 579.8	.4989	65.6	8.75
2	22.021611	2 398 582.5	1.6387	45.8	6.31
3	22.129270	2 289 451.4	2.7213	34.8	4.86
4	22.279117	2 188 625.1	3.7749	27.8	3.93
5	22.4703055	2 094 479.4	4.8121	23.1	3.28
7	22.972348	1 923 054.5	6.8599	17.1	2.45
10	24.004273	1 701 776.5	9.8995	12.3	1.75
20	29.347181	1 194 842.4	19.9501	6.4	.87
30	36.550554	910 182.00	29.9683	4.4	.55
40	44.717172	741 612.9	39.9782	3.4	.38
50	53.397874	636 917.5	49.9846	2.9	.27
70	71.502669	529 571.5	69.9933	2.3	.12
90	90	$5 \cdot 10^5$	90	2.2	0

TABLE B-32.- $h_f = 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	30.146297	3 798 160.3	-0.7920	108.8	13.82
.1	30.146448	3 782 316.7	-.6527	102.4	13.14
.2	30.146898	3 766 906.5	-.5166	96.7	12.51
.3	30.147650	3 751 883.3	-.3834	91.5	11.93
.4	30.148702	3 737 207.9	-.2529	86.8	11.39
.5	30.150054	3 722 846.2	-.1246	82.4	10.90
.7	30.153659	3 694 951.4	.1260	74.9	10.02
1	30.161320	3 654 845.6	.4897	65.7	8.91
2	30.206342	3 531 641.3	1.6330	45.9	6.41
3	30.281229	3 418 864.5	2.7175	34.8	4.93
4	30.385759	3 313 001.4	3.7721	27.8	3.98
5	30.519624	3 212 492.3	4.8100	23.1	3.32
7	30.873737	3 024 687.0	6.8585	17.1	2.47
10	31.612952	2 771 181.2	9.8986	12.3	1.77
20	35.64972	2 124 054.3	19.9497	6.4	.88
30	41.505517	1 703 468.0	29.9681	4.4	.56
40	48.514247	1 429 217.4	39.9780	3.4	.38
50	56.230860	1 248 508.9	49.9845	2.9	.27
70	72.796853	1 054 824.7	69.9933	2.3	.12
90	90	10^6	90	2.2	0

TABLE B-33.- $h_f = 2 \cdot 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	40.400588	5 522 895.2	-0.8049	109.2	14.05
.1	40.400690	5 507 046.3	-.6646	102.7	13.35
.2	40.400998	5 491 620.2	-.5277	96.9	12.70
.3	40.401510	5 476 570.5	-.3938	91.7	12.11
.4	40.402228	5 461 857.9	-.2625	86.9	11.56
.5	40.403151	5 447 448.45	-.1336	82.6	11.06
.7	40.405612	5 419 426.1	.1181	75.0	10.16
1	40.410840	5 379 049.6	.4830	65.7	9.02
2	40.441580	5 254 255.6	1.6291	45.9	6.47
3	40.492762	5 138 841.5	2.7149	34.8	4.98
4	40.564303	5 029 312.1	3.7703	27.8	4.01
5	40.656095	4 924 132.5	4.8087	23.1	3.34
7	40.899838	4 724 105.9	6.8576	17.1	2.48
10	41.412889	4 445 699.8	9.8981	12.3	1.78
20	44.307141	3 676 036.7	19.9495	6.4	.88
30	48.737809	3 111 878.4	29.9680	4.4	.56
40	54.312283	2 706 900.7	39.9779	3.4	.38
50	60.692072	2 420 931.4	49.9845	2.9	.27
70	74.902690	2 095 734.7	69.9933	2.3	.12
90	90	$2 \cdot 10^6$	90	2.2	0

TABLE B-34.- $h_f = 5 \cdot 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	55.892733	9 513 772.0	-0.8169	109.4	14.26
.1	55.892792	9 497 918.3	-.6757	102.9	13.54
.2	55.892970	9 482 477.7	-.5380	97.1	12.88
.3	55.893265	9 467 403.9	-.4033	91.8	12.27
.4	55.893679	9 452 657.4	-.2714	87.1	11.72
.5	55.894211	9 438 204.5	-.1418	82.7	11.20
.7	55.895629	9 410 066.1	.1109	75.1	10.28
1	55.898643	9 369 443.0	.4771	65.8	9.13
2	55.916367	9 243 200.2	1.6256	45.9	6.53
3	55.945894	9 125 377.3	2.7126	34.8	5.02
4	55.987202	9 012 487.5	3.7688	27.8	4.04
5	56.040261	8 903 008.2	4.8075	23.1	3.36
7	56.181472	8 691 628.3	6.8569	17.1	2.50
10	56.480205	8 389 626.4	9.8977	12.3	1.78
20	58.201940	7 494 273.3	19.9494	6.4	.88
30	60.946993	6 760 475.6	29.9679	4.4	.56
40	64.560688	6 177 029.9	39.9779	3.4	.39
50	68.872822	5 728 513.6	49.9844	2.9	.27
70	78.942983	5 173 693.3	69.9932	2.3	.12
90	90	$5 \cdot 10^6$	90	2.2	0

TABLE B-35.- $h_f = 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	67.073028	15 177 098.6	-0.8231	109.6	14.36
.1	67.073065	15 161 242.5	-.6814	103.1	13.64
.2	67.073176	15 145 794.5	-.5432	97.2	12.97
.3	67.073360	15 130 708.3	-.4082	91.9	12.36
.4	67.073619	15 115 944.6	-.2759	87.2	11.80
.5	67.073951	15 101 469.4	-.1461	82.8	11.28
.7	67.074837	15 073 271.8	.1072	75.2	10.35
1	67.076719	15 032 522.7	.4741	65.8	9.18
2	67.087790	14 905 539.3	1.6239	45.9	6.56
3	67.106236	14 786 483.7	2.7115	34.8	5.03
4	67.132049	14 671 871.8	3.7680	27.8	4.05
5	67.165216	14 560 184.2	4.8070	23.1	3.37
7	67.253548	14 342 949.1	6.8566	17.1	2.50
10	67.440708	14 028 663.5	9.8975	12.3	1.79
20	68.526900	13 065 199.9	19.9493	6.4	.88
30	70.283477	12 231 983.1	29.9679	4.4	.56
40	72.637367	11 533 979.4	39.9779	3.4	.39
50	75.498653	10 971 543.9	49.9844	2.9	.27
70	82.343340	10 238 884.9	69.9932	2.3	.12
90	90	10^7	90	2.2	0

TABLE B-36.- $h_f = 2 \cdot 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	76.002726	25 687 698.2	-0.8273	109.7	14.44
.1	76.002748	25 671 840.4	-.6853	103.2	13.71
.2	76.002813	25 656 387.3	-.5468	97.3	13.03
.3	76.002922	25 641 292.7	-.4115	92.0	12.42
.4	76.003074	25 626 517.1	-.2790	87.2	11.85
.5	76.003270	25 612 026.8	-.1489	82.8	11.33
.7	76.003792	25 583 788.7	.1047	75.2	10.39
1	76.004901	25 542 953.5	.4720	65.9	9.22
2	76.011426	25 415 464.1	1.6227	45.9	6.59
3	76.022299	25 295 566.0	2.7108	34.8	5.05
4	76.037515	25 179 776.3	3.7675	27.8	4.06
5	76.057069	25 066 577.6	4.8066	23.1	3.38
7	76.109161	24 845 329.7	6.8564	17.1	2.51
10	76.219608	24 522 598.3	9.8974	12.3	1.79
20	76.862493	23 511 630.5	19.9493	6.4	.89
30	77.908674	22 606 980.3	29.9679	4.4	.56
40	79.322102	21 823 492.7	39.9779	3.4	.39
50	81.055670	21 172 561.6	49.9844	2.9	.27
70	85.254705	20 294 491.8	69.9932	2.3	.12
90	90	$2 \cdot 10^7$	90	2.2	0

TABLE B-37.- $h_f = 5 \cdot 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	83.50199615	56 108 745.0	-0.8306	109.8	14.50
.1	83.50200609	56 092 894.1	-.6884	103.2	13.76
.2	83.50203591	56 077 437.9	-.5497	97.4	13.08
.3	83.50208560	56 062 336.8	-.4141	92.1	12.46
.4	83.50215518	56 047 552.0	-.2814	87.3	11.89
.5	83.50224464	56 033 049.8	-.1512	82.9	11.37
.7	83.50248319	56 004 780.2	.1027	75.2	10.42
1	83.50299009	55 963 878.0	.4704	65.9	9.24
2	83.50597160	55 835 994.1	1.6218	45.9	6.60
3	83.51093973	55 715 439.1	2.7102	34.8	5.06
4	83.51789292	55 598 731.0	3.7671	27.8	4.06
5	83.52682896	55 484 353.7	4.8063	23.1	3.38
7	83.55063767	55 259 974.0	6.8565	17.1	2.51
10	83.60113115	54 930 642.0	9.8973	12.3	1.79
20	83.89541180	53 882 319.7	19.9492	6.4	.89
30	84.37559065	52 920 761.6	29.9679	4.4	.56
40	85.02666131	52 067 992.4	39.9779	3.4	.39
50	85.82842712	51 343 960.4	49.9844	2.9	.27
70	87.78175881	50 342 660.9	69.9932	2.3	.12
90	90	$5 \cdot 10^7$	90	2.2	0

TABLE B-38.- $h_f = 10^8$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	86.56151026	106 279 432.7	-0.8319	109.8	14.52
.1	86.56151551	106 263 573.1	-.6896	103.3	13.78
.2	86.56153124	106 248 114.5	-.5508	97.4	13.10
.3	86.56155745	106 233 010.7	-.4152	92.1	12.48
.4	86.56159416	106 218 222.3	-.2824	87.3	11.91
.5	86.56164135	106 203 715.4	-.1521	82.9	11.38
.7	86.56176719	106 175 433.2	.1020	75.2	10.44
1	86.56203459	106 134 504.3	.4698	65.9	9.25
2	86.56360741	106 006 463.7	1.6214	46.0	6.61
3	86.56622825	105 885 647.6	2.7100	34.8	5.06
4	86.56989629	105 768 574.4	3.7670	27.8	4.07
5	86.57461040	105 653 728.3	4.8062	23.1	3.38
7	86.58717071	105 428 103.1	6.8562	17.2	2.51
10	86.61381003	105 096 144.9	9.8973	12.3	1.79
20	86.76910373	104 032 929.8	19.9492	6.4	.89
30	87.02262786	103 048 583.8	29.9679	4.4	.56
40	87.36661781	102 167 909.5	39.9779	3.4	.39
50	87.79056030	101 414 239.7	49.9844	2.9	.27
70	88.82459056	100 362 524.1	69.9932	2.3	.12
90	90	10^8	90	2.2	0

TABLE B-39.- $\rho_M = 5 \cdot 10^4$ METERS, $h_i = 2000$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
-2	402.45	49 986.32	-2.05582	13.68	0.974
-1.5	839.59	49 986.77	-1.55460	13.23	.953
-1	1 276.79	49 987.20	-1.05341	12.80	.932
-.5	1 714.00	49 987.62	-.55226	12.38	.912
-.17*	2 002.56	49 987.88	-.22152	12.12	.899
0	2 151.20	49 988.01	-.05115	11.99	.893
.5	2 588.35	49 988.39	.44994	11.61	.874
1	3 025.42	49 988.75	.95100	11.25	.855
1.5	3 462.38	49 989.10	1.45202	10.90	.837
2	3 899.19	49 989.43	1.95302	10.57	.820
3	4 772.25	49 990.06	2.95493	9.94	.787
4	5 644.32	49 990.64	3.95674	9.36	.755
5	6 515.15	49 991.16	4.95845	8.84	.725
7	8 252.04	49 992.10	7.89979	7.90	.670
10	10 841.54	49 993.25	9.96574	6.75	.598

* E_M undergoes a sign reversal.

TABLE B-40.- $h_f = 0$ AND $h_i = 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
-30.2	-1.891328	3 544 430.01	-30.21264	47.5	0.221
-30.3	-3.202376	3 396 949.69	-30.30713	33.1	.124
-30.4	-4.117712	3 300 912.24	-30.40515	27.2	.090
-30.5	-4.866020	3 225 681.22	-30.50408	23.6	.071
-30.6	-5.515705	3 162 474.24	-30.60340	21.2	.059
-30.8	-6.631702	3 058 060.34	-30.80258	18.0	.045
-31	-7.591112	2 972 196.24	-31.00210	15.9	.037
-31.5	-9.598356	2 803 315.09	-31.50145	12.8	.025
-32	-11.277798	2 672 450.62	-32.00113	11.0	.020
-32.5	-12.759550	2 564 376.66	-32.50093	9.8	.016
-33	-14.106058	2 471 851.13	-33.00079	8.9	.014
-34	-16.522814	2 318 513.57	-34.00062	7.6	.011
-35	-18.688456	2 193 992.22	-35.00051	6.8	.009
-37	-22.549384	1 998 977.52	-37.00039	5.7	.007
-40	-27.642368	1 786 927.40	-40.00029	4.7	.005
-45	-35.144319	1 548 038.17	-45.00020	3.8	.003
-50	-41.984534	1 387 165.34	-50.00015	3.3	.003
-55	-48.448805	1 271 870.69	-55.00012	2.9	.002
-60	-54.675642	1 186 633.73	-60.00009	2.7	.002
-70	-66.701891	1 075 474.08	-70.00005	2.4	.001
-90	-90	10^6	-90	2.2	0

APPENDIX C

This appendix contains tables of refraction corrections for

$N_o = 0.000255$

$H_s = 7892$ meters

$R_o = 6\ 378\ 165$ meters

$h_i = 0$ unless otherwise specified

The tables C-1 through C-40 were generated by using the set of three nonlinear differential equations to obtain the ray path. An integration step size of 5000 meters was used. The integrator was a fourth-order Runge-Kutta-Gill integrator.

The following definitions are used.

N_o = modulus of refraction at zero altitude above R_o

H_s = atmospheric scale height

R_o = radius of Earth, value is not critical

h_i = initial altitude of ray path above R_o

h_f = final altitude of ray path above R_o

E_{Mi} = initial measured elevation angle of ray path

E_{Mf} = final elevation angle of ray path

ρ_M = measured range = $\int n ds$

ρ = geometric, straight-line, value of range

E = geometric elevation angle

TABLE C-1.- $\rho_M = 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0 .0622	999.75	-0.00093	0.25	0.016
.1	1.807	999.75	.09907	.25	.016
.2	3.55	999.75	.19907	.25	.016
.3	5.30	999.75	.29908	.25	.016
.4	7.04	999.75	.39908	.25	.016
.5	8.79	999.75	.49908	.25	.016
.7	12.28	999.75	.69908	.25	.016
1	17.51	999.75	.99908	.25	.016
2	34.95	999.75	1.99908	.25	.016
3	52.38	999.75	2.99908	.25	.016
4	69.80	999.75	3.99908	.25	.016
5	87.20	999.75	4.99908	.25	.016
7	121.90	999.75	6.99909	.25	.016
10	173.66	999.75	9.99910	.25	.016
20	341.99	999.75	19.99914	.25	.015
30	499.92	999.75	29.99922	.25	.014
40	642.67	999.76	39.99931	.24	.012
50	765.88	999.76	49.99942	.24	.010
70	939.47	999.76	69.99970	.24	.005
90	999.76	999.76	90	.239	0

TABLE C-2.- $\rho_M = 2 \cdot 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.249	1999.49	-0.00185	0.51	0.032
.1	3.74	1999.49	.09815	.51	.032
.2	7.23	1999.49	.19815	.51	.032
.3	10.72	1999.49	.29815	.51	.032
.4	14.21	1999.49	.29815	.51	.032
.5	17.70	1999.49	.49815	.51	.032
.7	24.68	1999.49	.69815	.51	.032
1	35.14	1999.49	.99815	.51	.032
2	70.03	1999.49	1.99816	.51	.032
3	104.89	1999.49	2.99816	.51	.032
4	139.73	1999.49	3.99816	.51	.032
5	174.51	1999.50	4.99817	.50	.032
7	243.92	1999.50	6.99818	.50	.032
10	347.45	1999.50	9.99820	.50	.031
20	684.09	1999.51	19.99831	.49	.029
30	999.95	1999.52	29.99846	.48	.026
40	1285.42	1999.53	39.99866	.47	.023
50	1531.84	1999.54	49.99888	.46	.019
70	1878.99	1999.55	69.9941	.45	.010
90	1999.55	1999.55	90	.450	0

TABLE C-3.- $\rho_M = 5 \cdot 10^3$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1.555	4998.73	-0.00463	1.27	0.081
.1	10.28	4998.73	.09538	1.27	.081
.2	19.00	4998.73	.19538	1.27	.081
.3	27.73	4998.73	.29538	1.27	.081
.4	36.45	4998.73	.39538	1.27	.081
.5	45.18	4998.73	.49538	1.27	.081
.7	62.63	4998.73	.69539	1.27	.081
1	88.80	4998.73	.99539	1.27	.080
2	176.01	4998.74	1.99541	1.26	.080
3	263.17	4998.75	2.99543	1.25	.080
4	350.25	4998.75	3.99545	1.25	.079
5	437.22	4998.76	4.99548	1.24	.079
7	610.74	4998.77	6.99552	1.23	.078
10	869.55	4998.79	9.99561	1.21	.077
20	1711.11	4998.85	19.99595	1.15	.071
30	2500.65	4998.91	29.99639	1.09	.063
40	3214.21	4998.95	39.99689	1.05	.054
50	3830.11	4998.99	49.99745	1.01	.044
70	4697.75	4999.04	69.99869	.96	.023
90	4999.06	4999.06	90	.944	0

TABLE C-4.- $\rho_M = 10^4$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	6.22	9997.45	-0.00925	2.55	0.161
.1	23.67	9997.45	.09076	2.55	.161
.2	41.12	9997.46	.19076	2.54	.161
.3	58.57	9997.46	.29077	2.54	.161
.4	76.02	9997.46	.39078	2.54	.161
.5	93.47	9997.47	.49078	2.53	.161
.7	128.37	9997.47	.69080	2.53	.161
1	180.71	9997.48	.99082	2.52	.160
2	355.14	9997.51	1.99089	2.49	.159
3	529.47	9997.53	2.99096	2.47	.158
4	703.63	9997.56	3.99104	2.44	.156
5	877.58	9997.59	4.99111	2.41	.155
7	1224.61	9997.64	4.99127	2.36	.152
10	1742.23	9997.71	9.99152	2.29	.148
20	3425.17	9997.93	19.99244	2.07	.132
30	5003.94	9998.11	29.99344	1.89	.114
40	6430.61	9998.26	39.99450	1.74	.096
50	7661.93	9998.37	49.99559	1.63	.077
70	9396.31	9998.51	69.99779	1.49	.039
90	9998.55	9998.55	90	1.446	0

TABLE C-5.- $\rho_M = 2 \cdot 10^4$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	24.89	19 994.91	-0.01849	5.09	0.323
.1	59.97	19 994.91	.08153	5.08	.322
.2	94.70	19 994.93	.18156	5.07	.322
.3	129.61	19 994.94	.28159	5.06	.321
.4	164.51	19 994.95	.38162	5.05	.321
.5	199.42	19 994.96	.48164	5.04	.320
.7	269.23	19 994.98	.68170	5.02	.319
1	373.93	19 995.02	.98178	4.98	.318
2	722.86	19 995.12	1.98205	4.88	.313
3	1071.56	19 995.23	2.98232	4.77	.309
4	1419.93	19 995.33	3.98259	4.67	.304
5	1767.86	19 995.43	4.98286	4.57	.299
7	2461.97	19 995.62	6.98340	4.38	.290
10	3497.20	19 995.88	9.98419	4.12	.276
20	6862.54	19 996.59	19.98672	3.41	.232
30	10 018.72	19 997.11	29.98905	2.89	.191
40	12 870.14	19 997.48	39.99119	2.52	.154
50	15 330.56	19 997.75	49.99315	2.25	.119
70	18 795.29	19 997.06	69.99671	1.94	.057
90	19 998.15	19 998.15	90	1.853	0

TABLE C-6.- $\rho_M = 5 \cdot 10^4$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	155.65	49 987.33	-0.04611	12.67	0.805
.1	243.04	49 987.40	.05406	12.60	.802
.2	330.43	49 987.47	.15423	12.53	.799
.3	417.82	49 987.54	.25440	12.46	.796
.4	505.20	49 987.61	.35456	12.39	.793
.5	592.58	49 987.67	.45473	12.33	.790
.7	767.34	49 987.81	.65506	12.19	.784
1	1029.44	49 988.00	.95555	12.00	.776
2	1902.81	49 988.63	1.95713	11.37	.748
3	2775.49	49 989.21	2.95864	10.79	.722
4	3647.22	49 989.75	3.96008	10.25	.697
5	4517.74	49 990.26	4.96146	9.74	.673
7	6254.09	49 991.16	6.96403	8.84	.628
10	8842.93	49 992.30	9.96749	7.70	.567
20	17 252.42	49 994.81	19.97632	5.19	.413
30	25 131.20	49 996.15	29.98238	3.85	.308
40	32 243.05	49 996.93	39.98681	3.07	.230
50	38 375.28	49 997.40	49.99026	2.60	.170
70	47 004.07	49 997.86	69.99558	2.14	.077
90	49 997.99	49 997.99	90	2.009	0

TABLE C-7.- $\rho_M = 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	624.16	99 975.12	-0.09132	24.88	1.594
.1	799.79	99 975.39	.00934	24.61	1.582
.2	975.41	99 975.66	.11000	24.34	1.571
.3	1151.01	99 975.92	.21065	24.08	1.559
.4	1326.59	99 976.18	.31130	23.82	1.548
.5	1502.16	99 976.44	.41193	23.56	1.537
.7	1853.24	99 976.93	.61319	23.06	1.515
1	2379.71	99 977.66	.91502	22.34	1.483
2	4133.27	99 979.85	1.92073	20.15	1.384
3	5884.41	99 981.75	2.92589	18.25	1.294
4	7632.70	99 983.41	3.93056	16.59	1.212
5	9377.69	99 984.86	4.93480	15.14	1.138
7	12 855.97	99 987.23	6.94217	12.77	1.009
10	18 037.14	99 989.84	9.95103	10.16	.855
20	34 838.13	99 994.23	19.96915	5.77	.538
30	50 549.42	99 995.99	29.97875	4.01	.371
40	64 711.97	99 996.87	39.98476	3.13	.266
50	76 910.29	99 997.37	49.98902	2.63	.192
70	94 054.75	99 997.86	69.99513	2.14	.085
90	99 997.99	99 997.99	90	2.013	0

TABLE C-8.- $\rho_M = 2 \cdot 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	2520.07	100 053.64	-0.17585	46.36	3.069
.1	2877.58	199 954.62	-.07337	45.38	3.026
.2	3234.90	199 955.57	.02906	44.43	2.983
.3	3592.01	199 956.49	.13143	43.51	2.942
.4	3948.93	199 957.38	.23376	42.62	2.901
.5	4305.67	199 958.25	.33603	41.75	2.862
.7	5018.57	199 959.91	.54045	40.09	2.785
1	6086.58	199 962.23	.84673	37.77	2.675
2	9635.70	199 968.71	1.86501	31.29	2.354
3	13 169.55	199 973.67	2.88021	26.33	2.091
4	16 689.45	199 977.51	3.89275	22.49	1.872
5	20 196.18	199 980.53	4.90326	19.47	1.688
7	27 171.29	199 984.87	6.91974	15.13	1.401
10	37 535.50	199 988.85	9.93685	11.15	1.102
20	71 026.41	199 994.16	19.96474	5.84	.615
30	102 243.87	199 995.99	29.97678	4.01	.405
40	130 316.23	199 996.87	39.98370	3.13	.285
50	154 446.50	199 997.37	49.98839	2.63	.203
70	188 236.90	199 997.86	69.99491	2.14	.089
90	199 997.99	199 997.99	90	2.013	0

TABLE C-9.- $\rho_M = 5 \cdot 10^5$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	16 492.08	499 922.16	-0.35284	77.84	6.185
.1	17 451.96	499 925.37	-.24256	74.63	5.979
.2	18 407.51	499 928.38	-.13276	71.62	5.808
.3	19 359.01	499 931.19	-.02342	68.81	5.645
.4	20 306.66	499 933.84	.08550	66.16	5.489
.5	21 250.70	499 936.33	.19403	63.67	5.340
.7	23 128.72	499 940.87	.40997	59.13	5.062
1	25 923.18	499 946.77	.73141	53.23	4.688
2	35 085.65	499 960.76	1.78658	39.24	3.725
3	44 081.96	499 969.44	2.82469	30.56	3.060
4	52 965.94	499 975.20	3.85219	24.80	2.580
5	61 767.29	499 979.24	4.87276	20.76	2.221
7	79 182.30	498 984.45	6.90120	15.55	1.724
10	104 926.50	499 988.77	9.92672	11.23	1.279
20	187 537.30	499 994.16	19.96204	5.84	.663
30	263 949.78	499 995.99	29.97560	4.01	.426
40	332 224.02	499 996.87	39.98307	3.13	.296
50	390 591.22	499 997.37	49.98802	2.63	.206
70	471 964.69	499 997.86	69.99478	2.14	.091
90	499 997.99	499 997.99	90	2.013	0

TABLE C-10.- $\rho_M = 10^6$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	69 959.45	999 917.35	-0.46049	82.65	8.037
.1	71 985.55	999 921.19	-.34314	78.81	7.734
.2	73 992.37	999 924.74	-.22686	75.26	7.450
.3	75 981.50	999 928.03	-.11157	71.97	7.183
.4	77 954.34	999 931.08	.00281	68.92	6.932
.5	79 912.14	999 933.91	.11636	66.09	6.696
.7	83 787.08	999 939.02	.34119	60.98	6.262
1	89 511.74	999 945.50	.67362	54.50	5.696
2	108 044.84	999 960.38	1.75209	39.62	4.327
3	126 032.53	999 969.31	2.80240	30.69	3.449
4	143 684.43	999 975.15	3.83687	24.85	2.847
5	161 103.93	999 979.22	4.86171	20.78	2.414
7	195 440.26	999 984.44	6.89478	15.56	1.836
10	245 975.61	999 988.76	9.92331	11.24	1.338
20	406 799.36	999 994.16	19.96114	5.84	.678
30	553 958.65	999 995.99	29.97520	4.01	.433
40	684 245.51	999 996.87	39.98285	3.13	.299
50	794 780.03	999 997.37	49.98789	2.63	.211
70	947 651.47	999 997.86	69.99473	2.14	.092
90	999 997.99	999 997.99	90	2.013	0

TABLE C-11.- $\rho_M = 2 \cdot 10^6$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	288 979.5	1 999 916.4	-0.5162	83.6	9.01
.1	293 025.8	1 999 920.4	-.3950	79.6	8.64
.2	297 022.1	1 999 924.0	-.2753	76.0	8.29
.3	300 972.8	1 999 927.4	-.1568	72.6	7.97
.4	304 881.8	1 999 930.5	-.0395	69.5	7.67
.5	308 752.5	1 999 933.4	.0767	66.6	7.39
.7	316 390.8	1 999 938.6	.3062	61.4	6.87
1	327 628.3	1 999 945.2	.6443	54.8	6.21
2	363 729.9	1 999 960.3	1.7348	39.7	4.63
3	398 503.4	1 999 969.3	2.7912	30.7	3.64
4	432 464.0	1 999 975.1	3.8292	24.9	2.98
5	465 858.5	1 999 979.2	4.8562	20.8	2.51
7	531 407.3	1 999 984.4	6.8916	15.6	1.89
10	627 316.1	1 999 988.8	9.9216	11.2	1.37
20	928 703.7	1 999 994.2	19.9607	5.8	.69
30	1 199 937.5	1 999 996.0	29.9750	4.0	.44
40	1 436 838.8	1 999 996.9	39.9827	3.1	.30
40	1 635 654.3	1 999 997.4	49.9878	2.6	.21
70	1 907 618.4	1 999 997.9	69.9947	2.1	.09
90	1 999 998.0	1 999 998.0	90	2.013	0

TABLE C-12.- $\rho_M = 5 \cdot 10^6$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1 688 329.6	4 999 915.8	-0.5496	84.2	9.59
.1	1 696 847.1	4 999 919.9	-.4261	80.1	9.18
.2	1 705 243.2	4 999 923.6	-.3043	76.4	8.80
.3	1 713 528.8	4 999 927.0	-.1839	73.0	8.45
.4	1 721 713.2	4 999 930.2	-.0649	69.8	8.11
.5	1 729 805.1	4 999 933.1	.0529	66.9	7.80
.7	1 745 740.3	4 999 938.4	.2852	61.6	7.24
1	1 769 112.2	4 999 945.1	.6268	54.9	6.51
2	1 843 746.7	4 999 960.2	1.7243	39.8	4.81
3	1 915 156.6	4 999 969.2	2.7845	30.8	3.76
4	1 984 556.3	4 999 975.1	3.8246	24.9	3.06
5	2 052 524.0	4 999 979.2	4.8529	20.8	2.57
7	2 185 239.8	4 999 984.4	6.8897	15.6	1.93
10	2 377 938.3	4 999 988.8	9.9206	11.2	1.39
20	2 973 524.6	4 999 994.2	19.9604	5.8	.69
30	3 498 450.5	4 999 996.0	29.9749	4.0	.44
40	3 949 671.8	4 999 996.9	39.9827	3.1	.30
50	4 323 787.0	4 999 997.4	49.9878	2.6	.21
70	4 829 603.2	4 999 997.9	69.9947	2.1	.09
90	4 999 998.0	4 999 998.0	90	2.013	0

TABLE C-13.- $\rho_M = 10^7$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	5 429 919.9	9 999 915.6	-0.5608	84.4	9.79
.1	5 441 629.6	9 999 919.7	-.4365	80.3	9.36
.2	5 453 167.7	9 999 923.5	-.3140	76.5	8.97
.3	5 464 549.4	9 999 926.9	-.1930	73.1	8.60
.4	5 475 788.6	9 999 930.1	-.0734	69.9	8.26
.5	5 486 897.3	9 999 933.0	.0449	67.0	7.94
.7	5 508 765.0	9 999 938.3	.2782	61.7	7.36
1	5 540 821.1	9 999 945.0	.6209	55.0	6.62
2	5 643 106.7	9 999 960.2	1.7209	39.8	4.87
3	5 740 928.6	9 999 969.2	2.7823	30.8	3.80
4	5 836 000.7	9 999 975.1	3.8231	24.9	3.09
5	5 929 134.3	9 999 979.2	4.8518	20.8	2.59
7	6 111 087.2	9 999 984.4	6.8890	15.6	1.94
10	6 375 539.5	9 999 988.8	9.9202	11.2	1.39
20	7 194 861.4	9 999 994.2	19.9603	5.8	.69
30	7 919 178.5	9 999 996.0	29.9748	4.0	.44
40	8 543 209.9	9 999 996.9	39.9827	3.1	.30
50	9 061 477.9	9 999 997.4	49.9878	2.6	.21
70	9 763 308.6	9 999 997.9	69.9947	2.1	.09
90	9 999 998.0	9 999 998.0	90	2.013	0

TABLE C-14.- $\rho_M = 2 \cdot 10^7$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	14 554 009.7	19 999 915.6	-0.5663	84.4	9.88
.1	14 567 263.9	19 999 919.6	-.4417	80.4	9.45
.2	14 580 325.3	19 999 923.4	-.3188	76.6	9.06
.3	14 593 211.6	19 999 926.8	-.1975	73.2	8.68
.4	14 605 938.3	19 999 930.0	-.0776	70.0	8.34
.5	14 618 519.3	19 999 933.0	.0410	67.0	8.01
.7	14 643 291.6	19 999 938.3	.2747	61.7	7.42
1	14 679 623.1	19 999 945.0	.6180	55.0	6.67
2	14 795 716.5	19 999 960.2	1.7191	39.8	4.90
3	14 907 015.8	19 999 969.2	2.7812	30.8	3.82
4	15 015 462.9	19 999 975.1	3.8223	24.9	3.10
5	15 121 970.2	19 999 979.2	4.8512	20.8	2.60
7	15 330 829.8	19 999 984.4	6.887	15.6	1.94
10	15 636 199.6	19 999 988.8	9.9201	11.2	1.39
20	16 595 144.3	19 999 994.2	19.9603	5.8	.69
30	17 457 669.8	19 999 996.0	29.9748	4.0	.44
40	18 210 732.0	19 999 996.9	39.9826	3.1	.30
50	18 842 544.1	19 999 997.4	49.9878	2.6	.21
70	19 706 569.5	19 999 997.9	69.9947	2.1	.09
90	19 999 998.0	19 999 998.0	90	2.013	0

TABLE C-15.- $\rho_M = 5 \cdot 10^7$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	43 963 975.8	49 999 915.5	-0.5697	84.5	9.94
.1	43 977 781.3	49 999 919.6	-.4448	80.4	9.51
.2	43 991 389.0	49 999 923.3	-.3217	76.7	9.11
.3	44 004 817.2	49 999 926.8	-.2002	73.2	8.73
.4	44 018 082.1	49 999 930.0	-.0802	70.0	8.38
.5	44 031 198.2	49 999 933.0	.0386	67.0	8.05
.7	44 057 033.4	49 999 938.3	.2726	61.7	7.46
1	44 094 947.3	49 999 945.0	.6162	55.0	6.70
2	44 216 302.2	49 999 960.2	1.7181	39.8	4.92
3	44 332 962.0	49 999 969.2	2.7805	30.8	3.83
4	44 446 944.2	49 999 975.1	3.8218	24.9	3.11
5	44 559 192.6	49 999 979.2	4.8509	20.8	2.60
7	44 780 192.8	49 999 984.4	6.8885	15.6	1.95
10	45 105 405.7	49 999 988.8	9.9200	11.2	1.40
20	46 142 408.3	49 999 994.2	19.9603	5.8	.69
30	47 094 687.8	49 999 996.0	29.9748	4.0	.44
40	47 940 461.1	49 999 996.9	39.9826	3.1	.30
50	48 659 903.1	49 999 997.4	49.9878	2.6	.21
70	49 657 644.9	49 999 997.9	69.9947	2.1	.09
90	49 999 998.0	49 999 998.0	90	2.013	0

TABLE C-16.- $\rho_M = 10^8$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	93 761 519.8	99 999 915.5	-0.5708	84.5	9.96
.1	93 775 409.9	99 999 919.6	-.4459	80.4	9.53
.2	93 789 102.0	99 999 923.3	-.3227	76.7	9.12
.3	93 802 614.8	99 999 926.8	-.2011	73.2	8.75
.4	93 815 964.6	99 999 930.0	-.0810	70.0	8.40
.5	93 829 165.7	99 999 933.0	.0378	67.0	8.07
.7	93 855 172.3	99 999 938.3	.2719	61.7	7.47
1	93 893 347.0	99 999 945.0	.6156	55.0	6.71
2	94 015 617.3	99 999 960.2	1.7178	39.8	4.93
3	94 133 281.4	99 999 969.2	2.7803	30.8	3.83
4	94 248 367.7	99 999 975.1	3.8217	24.9	3.11
5	94 361 824.0	99 999 979.2	4.8508	20.8	2.60
7	94 585 555.0	99 999 984.4	6.8884	15.6	1.95
10	94 915 637.3	99 999 988.8	9.9199	11.2	1.40
20	95 974 852.0	99 999 994.1	19.9602	5.9	.69
30	96 956 296.5	99 999 996.0	29.9748	4.0	.44
40	97 834 815.4	99 999 996.9	39.9826	3.1	.30
50	98 587 052.6	99 999 997.4	49.9878	2.6	.21
70	99 637 602.7	99 999 997.8	69.9947	2.2	.09
90	99 999 998.0	99 999 998.0	90	2.013	0

TABLE C-17.- $h_f = 10$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.090412	12 675.06	-0.01173	3.23	0.205
.1	.134812	4 880.18	.09549	1.24	.079
.2	.219486	2 731.72	.19747	.70	.044
.3	.313328	1 868.37	.29827	.48	.030
.4	.410090	1 414.57	.39869	.36	.023
.5	.508108	1 136.71	.49895	.29	.018
.7	.705814	815.15	.69925	.21	.013
1	1.004078	571.82	.99947	.15	.009
2	2.002042	286.39	1.99974	.07	.005
3	3.001361	191.03	2.99982	.05	.003
4	4.001020	143.34	3.99987	.04	.002
5	5.000815	114.73	4.99989	.03	.002
7	7.000581	82.05	6.99992	.02	.001
10	10.000405	57.59	9.99995	.01	.001
20	20.000196	29.24	19.99997	.01	0
30	30.000124	20.00	29.99998	.01	0
40	40.000085	15.56	39.99999	0	0
50	50.000060	13.05	49.99999	0	0
70	70.000026	10.64	70.00000	0	0
90	90	10	90	0	0

TABLE C-18.- $h_f = 20$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.127872	17 924.76	-0.01658	4.57	0.289
.1	.162331	8 736.65	.09192	2.23	.141
.2	.237384	5 239.92	.19515	1.33	.085
.3	.326115	3 660.43	.29662	.93	.059
.4	.419942	2 795.15	.39742	.71	.045
.5	.516092	2 255.57	.49791	.57	.036
.7	.711583	1 623.63	.69850	.41	.026
1	1.008142	1 141.33	.99894	.29	.018
2	2.004082	572.49	1.99947	.15	.009
3	3.002721	381.97	2.99965	.10	.006
4	4.002040	286.64	3.99974	.07	.005
5	5.001631	229.44	4.99979	.06	.004
7	7.001162	164.10	6.99985	.04	.003
10	10.000809	115.17	9.99990	.03	.002
20	20.000392	58.48	19.99995	.01	.001
30	30.000247	40.00	29.99997	.01	.001
40	40.000170	31.11	39.99998	.01	0
50	50.000120	26.11	49.99998	.01	0
70	70.000052	21.28	69.99999	.01	0
90	90	20	90	.005	0

TABLE C-19.- $h_f = 50$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.202233	28 339.24	-0.02620	7.21	0.457
.1	.225606	17 598.51	.08374	4.48	.284
.2	.284426	11 828.57	.18908	3.01	.191
.3	.361798	8 657.86	.29200	2.20	.140
.4	.448216	6 755.02	.39376	1.72	.109
.5	.539349	5 512.79	.49491	1.40	.089
.7	.728626	4 010.67	.69630	1.02	.065
1	1.020242	2 836.24	.99738	.72	.046
2	2.010194	1 429.05	1.99868	.36	.023
3	3.006802	954.29	2.99912	.24	.015
4	4.005101	716.32	3.99934	.18	.012
5	5.004078	573.45	4.99947	.15	.009
7	7.002906	410.19	6.99962	.10	.007
10	10.002024	287.91	9.99974	.07	.005
20	20.000981	146.19	19.99987	.04	.002
30	30.000618	100.00	29.99992	.03	.001
40	40.000425	77.79	39.99994	.02	.001
50	50.000299	65.27	49.99996	.02	.001
70	70.000130	53.21	69.99998	.01	0
90	90	50	90	.01	0

TABLE C-20.- $h_f = 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.286116	40 072.35	-0.03701	10.18	0.646
.1	.303088	28 436.19	.07377	7.21	.458
.2	.349088	20 872.58	.18075	5.29	.336
.3	.414562	16 038.10	.28522	4.07	.258
.4	.491794	12 850.40	.38816	3.26	.207
.5	.576073	10 649.61	.49019	2.70	.171
.7	.756213	7 869.52	.69275	1.99	.127
1	1.040122	5 617.25	.99482	1.42	.090
2	2.020354	2 850.88	1.99737	.72	.046
3	3.013600	1 906.42	2.99825	.48	.031
4	4.010203	1 431.74	3.99868	.36	.023
5	5.008159	1 146.44	4.99895	.29	.018
7	7.005816	820.21	6.99925	.21	.013
10	10.004051	575.76	9.99948	.15	.009
20	20.001963	292.37	19.99975	.07	.004
30	30.001237	200.00	29.99984	.05	.003
40	40.000851	155.57	39.99989	.04	.002
50	50.000599	130.54	49.99992	.03	.001
70	70.000260	106.42	69.99997	.03	.001
90	90	10^2	90	.025	0

TABLE C-21.- $h_f = 2 \cdot 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.4049544	56 655.69	-0.05221	14.33	0.911
.1	.417119	44 348.54	.05920	11.20	.712
.2	.451650	35 184.41	.16766	8.88	.564
.3	.503971	28 514.37	.27381	7.19	.457
.4	.569197	23 651.39	.37828	5.96	.379
.5	.643416	20 046.75	.48160	5.05	.321
.7	.808690	15 192.47	.68606	3.83	.243
1	1.078875	11 025.43	.98988	2.78	.177
2	2.040569	5673.29	1.99480	1.43	.091
3	3.027183	3804.26	2.99651	.96	.061
4	4.020413	2859.84	3.99738	.72	.046
5	5.016331	2291.01	4.99791	.58	.037
7	7.011645	1639.75	6.99851	.41	.026
10	10.008113	1151.29	9.99896	.29	.018
20	20.003931	584.71	19.99950	.15	.009
30	30.002479	399.99	29.99968	.10	.006
40	40.001705	311.14	39.99978	.08	.004
50	50.001201	261.08	49.99985	.07	.003
70	70.000521	212.84	69.99993	.05	.001
90	90	200	90	.05	0

TABLE C-22.- $h_f = 5 \cdot 10^2$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.641802	89 509.49	-0.08197	22.38	1.431
.1	.649546	76 588.78	.03006	19.09	1.221
.2	.672241	65 782.06	.14005	16.36	1.046
.3	.708453	56 876.59	.24824	14.12	.903
.4	.756243	49 594.26	.35491	12.30	.787
.5	.813572	43 646.52	.46035	10.82	.692
.7	.949679	34 746.43	.66847	8.60	.550
1	1.188221	26 191.21	.97625	6.48	.415
2	2.100415	13 977.04	1.98734	3.45	.221
3	3.067822	9447.25	2.99145	2.33	.149
4	4.051079	7122.51	3.99356	1.76	.112
5	5.040919	5713.60	4.99484	1.41	.090
7	7.029215	4094.28	6.99632	1.01	.064
10	10.020365	2876.49	9.99743	.71	.045
20	20.009874	1461.56	19.99876	.36	.022
30	30.006225	999.91	29.99922	.25	.014
40	40.004284	777.83	39.99946	.19	.009
50	50.003016	652.69	49.99962	.16	.007
70	70.001308	532.09	69.99984	.13	.003
90	90	$5 \cdot 10^2$	90	.12	0

TABLE C-23.- $h_f = 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	0.911081	126 422.43	-0.11459	31.01	2.000
.1	.916552	113 190.02	-.00217	27.63	1.783
.2	.932773	101 495.20	.10869	24.69	1.594
.3	.959198	91 247.44	.21812	22.13	1.429
.4	.995015	82 323.22	.32628	19.92	1.287
.5	1.039253	74 580.42	.43332	18.01	1.164
.7	1.148924	62 056.08	.64465	14.95	.966
1	1.352769	48 745.19	.95661	11.72	.757
2	2.197665	27 311.45	1.97575	6.55	.423
3	3.135173	18 688.80	2.98343	4.48	.289
4	4.102282	14 155.81	3.98746	3.39	.219
5	5.082122	11 380.96	4.98994	2.73	.176
7	7.058749	8 171.56	6.99280	1.96	.126
10	10.040997	5 747.17	9.99498	1.38	.088
20	20.109892	2 922.42	19.99756	.70	.043
30	30.012544	1 999.62	29.99846	.48	.027
40	40.008632	1 555.58	39.99894	.37	.018
50	50.006078	1 305.35	49.99925	.31	.013
70	70.002636	1 064.17	69.99968	.25	.006
90	90	10 ³	90	.24	0

TABLE C-24.- $h_f = 2 \cdot 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	1.297538	178 348.47	-0.15845	42.14	2.765
.1	1.301385	164 897.63	-.04561	38.69	2.541
.2	1.312858	152 564.28	.06597	35.59	2.339
.3	1.331762	141 304.79	.17640	32.80	2.157
.4	1.357784	131 063.32	.28578	30.29	1.993
.5	1.390526	121 774.60	.39421	28.05	1.846
.7	1.474286	105 766.90	.60856	24.23	1.596
1	1.638119	87 078.82	.92508	19.84	1.308
2	2.383888	52 333.99	1.95528	11.84	.780
3	3.268343	36 594.81	2.96881	8.26	.544
4	4.204862	27 962.97	3.97621	6.31	.415
5	5.165204	22 578.85	4.98083	5.09	.335
7	7.118653	16 275.06	6.98623	3.67	.240
10	10.082979	11 470.87	9.99037	2.58	.168
20	20.040326	5842.01	19.99532	1.32	.082
30	30.025437	3998.48	29.99705	.90	.051
40	40.017506	3110.89	39.99797	.70	.035
50	50.012327	2610.58	49.99857	.59	.025
70	70.005347	2128.32	69.99938	.48	.011
90	90	2.103	90	.45	0

TABLE C-25.- $h_f = 5 \cdot 10^3$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	2.087679	280 144.53	-0.23527	59.61	4.106
.1	2.090072	266 505.67	-.12166	56.07	3.869
.2	2.097233	253 614.77	-.00910	52.82	3.649
.3	2.109115	241 447.49	.10250	49.83	3.447
.4	2.125637	229 978.01	.21321	47.07	3.260
.5	2.146694	219 179.02	.32311	44.53	3.087
.7	2.201860	199 476.97	.54076	40.02	2.779
1	2.314727	174 208.79	.86267	34.46	2.397
2	2.890784	117 591.88	1.90935	22.68	1.582
3	3.654370	86 310.56	2.93408	16.49	1.151
4	4.511244	67 455.62	3.94873	12.83	.895
5	5.417318	55 117.62	4.95825	10.46	.729
7	7.303198	40 178.09	6.96974	7.61	.528
10	10.213425	28 498.51	9.97872	5.39	.371
20	20.104226	14 583.27	19.98962	2.76	.181
30	30.065805	9990.25	29.99345	1.89	.114
40	40.045301	7775.03	39.99549	1.47	.079
50	50.031903	6525.54	49.99682	1.23	.055
70	70.013841	5320.66	69.99862	1.01	.024
90	90	$5 \cdot 10^3$	90	.94	0

TABLE C-26.- $h_f = 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	3.013162	392 694.29	-0.30362	71.82	5.299
.1	3.014819	378 965.96	-.18906	68.19	5.045
.2	3.019786	365 807.86	-.07554	64.85	4.809
.3	3.028046	353 199.58	.03705	61.75	4.589
.4	3.039572	341 121.71	.14878	58.88	4.385
.5	3.054327	329 555.51	.25972	56.21	4.194
.7	3.093331	307 885.75	.47952	51.42	3.848
1	3.174622	278 774.64	.80478	45.36	3.407
2	3.616000	205 784.19	1.86248	31.72	2.400
3	4.250982	158 895.04	2.89626	23.87	1.811
4	5.006435	127 752.64	3.91766	18.93	1.437
5	5.835759	106 132.35	4.93214	15.61	1.184
7	7.618000	78 693.05	6.95021	11.48	.869
10	10.439672	56 393.99	9.96473	8.19	.615
20	20.216510	29 091.13	19.98270	4.21	.302
30	30.136916	19 959.82	29.98906	2.89	.191
40	40.094309	15 542.41	39.99247	2.25	.131
50	50.066434	13 047.90	49.99470	1.89	.093
70	70.028828	10 640.83	69.99770	1.54	.040
90	90	10^4	90	1.45	0

TABLE C-27.- $h_f = 2 \cdot 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	4.358441	548 443.37	-0.37069	79.35	6.470
.1	4.359586	534 655.88	-.25480	75.64	6.192
.2	4.363019	521 320.53	-.13999	72.20	5.934
.3	4.368734	508 418.17	-.02617	69.02	5.693
.4	4.376722	495 931.46	.08675	66.07	5.467
.5	4.386697	483 844.54	.19885	63.32	5.256
.7	4.414189	460 812.53	.42087	58.37	4.872
1	4.471474	428 927.31	.74924	52.07	4.377
2	4.794610	342 281.76	1.81605	37.62	3.211
3	5.289484	279 232.75	2.85723	28.96	2.492
4	5.913132	232 911.38	3.88452	23.33	2.016
5	6.629309	198 283.06	4.90364	19.44	1.682
7	8.240228	151 203.02	6.92827	14.47	1.252
10	10.899663	110 412.94	9.94872	10.40	.895
20	20.450375	57 870.57	19.97463	5.38	.443
30	30.285753	39 833.28	29.98394	3.70	.280
40	40.197060	31 052.81	39.98894	2.88	.193
50	50.138890	26 082.44	49.99220	2.42	.136
70	70.060295	21 279.61	69.99662	1.97	.059
90	90	$2 \cdot 10^4$	90	1.85	0

TABLE C-28.- $h_f = 5 \cdot 10^4$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	7.033700	850 791.41	-0.44098	82.30	7.697
.1	7.034408	836 955.02	-.32295	78.46	7.382
.2	7.036529	823 472.86	-.20613	74.93	7.088
.3	7.040063	810 326.27	-.09042	71.66	6.814
.4	7.045008	797 498.68	.02429	68.62	6.557
.5	7.051360	784 975.27	.13807	65.80	6.317
.7	7.068273	760 789.18	.36318	60.72	5.879
1	7.104077	726 498.24	.69565	54.27	5.312
2	7.311144	626 857.62	1.77271	39.48	3.967
3	7.643800	545 661.03	2.82098	30.60	3.125
4	8.086560	479 126.06	3.85352	24.79	2.557
5	8.622480	424 382.89	4.87669	20.74	2.152
7	9.910918	341 412.12	6.90714	15.53	1.621
10	12.205289	260 223.20	9.93299	11.21	1.169
20	21.152898	142 347.23	19.96656	5.83	.584
30	30.738619	98 921.19	29.97879	4.01	.370
40	40.511163	77 384.31	39.98537	3.12	.255
50	50.360861	65 102.30	49.98969	2.26	.180
70	70.156862	53 183.00	69.99553	2.14	.078
90	90	$5 \cdot 10^4$	90	2.01	0

TABLE C-29.- $h_f = 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E, deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	9.997743	1 188 333.91	-0.47816	82.95	8.345
.1	9.998238	1 174 473.95	-.35855	79.05	8.003
.2	9.999723	1 160 920.98	-.24028	75.46	7.684
.3	10.002197	1 147 656.43	-.12321	72.13	7.386
.4	10.005660	1 134 663.87	-.00724	69.05	7.108
.5	10.010111	1 121 928.73	.10773	66.19	6.846
.7	10.021970	1 097 180.03	.33499	61.05	6.371
1	10.047124	1 061 703.25	.67021	54.53	5.756
2	10.1938305	955 343.77	1.75371	39.61	4.299
3	10.433759	863 831.46	2.80592	30.68	3.387
4	10.760673	784 420.68	3.84108	24.84	2.774
5	11.166937	715 284.24	4.86611	20.78	2.337
7	12.184254	602 289.25	6.89902	15.56	1.762
10	14.104461	479 892.75	9.92701	11.23	1.274
20	22.267773	277 507.39	19.96348	5.84	.637
30	31.474299	195 700.33	29.97682	4.01	.404
40	41.026004	153 952.59	39.98402	3.13	.279
50	50.726224	129 859.78	49.98873	2.63	.197
70	70.316359	106 312.44	69.99511	2.14	.085
90	90	10^5	90	2.01	0

TABLE C-30.- $h_f = 2 \cdot 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	14.106674	1 666 946.2	-0.5051	83.4	8.82
.1	14.107021	1 653 069.8	-.3841	79.5	8.45
.2	14.108063	1 639 467.5	-.2646	75.8	8.11
.3	14.109799	1 626 120.8	-.1464	72.5	7.79
.4	14.112229	1 613 013.2	-.0294	69.3	7.49
.5	14.115353	1 600 130.4	.0866	66.4	7.22
.7	14.123679	1 574 988.4	.3156	61.3	6.71
1	14.1413565	1 538 679.3	.6531	54.7	6.05
2	14.244897	1 427 515.2	1.7417	39.7	4.51
3	14.415812	1 328 326.0	2.7969	30.7	3.55
4	14.651742	1 238 803.5	3.8339	24.9	2.90
5	14.949608	1 157 634.7	4.8602	20.8	2.44
7	15.716334	1 016 892.2	6.8947	15.6	1.84
10	17.232683	850 026.0	9.9239	11.2	1.33
20	24.306734	530 413.4	19.9619	5.8	.66
30	32.869465	383 493.6	29.9758	4.0	.42
40	42.017251	304 798.6	39.9833	3.1	.29
50	51.434898	258 387.1	49.9883	2.6	.205
70	70.627641	212 415.3	69.9949	2.1	.09
90	90	$2 \cdot 10^5$	90	2.0	0

TABLE C-31.- $h_f = 5 \cdot 10^5$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	21.945056	2 634 152.1	-0.5296	83.8	9.24
.1	21.945273	2 620 261.2	-.4073	79.8	8.85
.2	21.945923	2 606 615.4	-.2865	76.1	8.49
.3	21.947006	2 593 196.0	-.1671	72.7	8.15
.4	21.948522	2 579 986.7	-.0490	69.6	7.84
.5	21.950470	2 566 973.0	.0679	66.7	7.54
.7	21.955667	2 541 482.6	.2987	61.4	7.00
1	21.966705	2 504 434.3	.6384	54.8	6.31
2	22.031522	2 388 960.2	1.7319	39.7	4.68
3	22.139127	2 282 728.6	2.7898	30.7	3.67
4	22.288901	2 183 630.9	3.8286	24.9	2.99
5	22.479997	2 090 610.5	4.8559	20.8	2.51
7	22.981804	1 920 534.9	6.8917	15.6	1.89
10	24.0132745	1 700 282.8	9.9219	11.2	1.36
20	29.354311	1 194 437.0	19.9610	5.8	.68
30	36.555962	910 034.1	29.9752	4.0	.43
40	44.721221	741 550.7	39.9829	3.1	.30
50	53.400853	636 889.8	49.9880	2.6	.21
70	71.504011	529 567.0	69.9948	2.1	.09
90	90	$5 \cdot 10^5$	90	2.0	0

TABLE C-32.- $h_f = 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	30.153200	3 769 817.0	-0.5423	84.0	9.47
.1	30.153351	3 755 918.7	-.4193	80.0	9.06
.2	30.153801	3 742 250.7	-.2978	76.3	8.69
.3	30.154552	3 728 794.4	-.1778	72.9	8.34
.4	30.155604	3 715 533.4	-.0591	69.7	8.01
.5	30.156956	3 702 453.3	.0584	66.8	7.71
.7	30.160560	3 676 785.7	.2902	61.5	7.15
1	30.168219	3 639 361.1	.6311	54.9	6.44
2	30.213229	3 521 682.4	1.7273	39.75	4.76
3	30.288095	3 411 809.0	2.7866	30.7	3.72
4	30.392596	3 307 679.8	3.8262	24.9	3.03
5	30.526425	3 208 302.7	4.8541	20.8	2.55
7	30.880442	3 021 863.4	6.8905	15.6	1.91
10	31.619465	2 769 414.6	9.9211	11.2	1.38
20	35.655306	2 123 487.6	19.9607	5.8	.69
30	41.510048	1 703 235.5	29.9750	4.0	.44
40	48.517793	1 429 112.3	39.9828	3.1	.30
50	56.233542	1 248 460.0	49.9879	2.6	.21
70	72.798094	1 054 816.4	69.9947	2.1	.09
90	90	10^6	90	2.0	0

TABLE C-33.- $h_f = 2 \cdot 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	40.405298	5 494 307.9	-0.5516	84.2	9.63
.1	40.405401	5 480 404.4	-.4280	80.1	9.21
.2	40.405708	5 466 720.5	-.3059	76.4	8.83
.3	40.406221	5 453 237.6	-.1854	73.0	8.47
.4	40.406939	5 439 939.4	-.0663	69.8	8.14
.5	40.407861	5 426 811.5	.0516	66.9	7.83
.7	40.410321	5 401 016.5	.2841	61.6	7.26
1	40.415549	5 363 321.4	.6260	54.9	6.53
2	40.446284	5 244 053.8	1.7240	39.8	4.82
3	40.497457	5 131 544.7	2.7844	30.8	3.76
4	40.568987	5 023 751.3	3.8246	24.9	3.06
5	40.660764	4 919 706.4	4.8529	20.8	2.57
7	40.904467	4 721 052.8	6.8897	15.6	1.92
10	41.417435	4 443 717.4	9.9207	11.2	1.38
20	44.311248	3 675 315.0	19.9605	5.8	.69
30	48.741327	3 111 548.2	29.9749	4.0	.44
40	54.315163	2 706 738.9	39.9827	3.1	.30
50	60.694322	2 420 851.7	49.9878	2.6	.21
70	74.903772	2 095 720.3	69.9947	2.1	.09
90	90	$2 \cdot 10^6$	90	2.0	0

TABLE C-34.- $h_f = 5 \cdot 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	55.895448	9 484 962.7	-0.5601	84.3	9.78
.1	55.895507	9 471 054.3	-.4359	80.3	9.35
.2	55.895685	9 457 355.9	-.3134	76.5	8.96
.3	55.895980	9 443 848.9	-.1924	73.1	8.59
.4	55.896394	9 430 516.9	-.0729	69.9	8.25
.5	55.896926	9 417 345.5	.04543	66.9	7.93
.7	55.898344	9 391 434.4	.2786	61.7	7.35
1	55.901357	9 353 492.8	.6213	55.0	6.61
2	55.919080	9 232 776.9	1.7212	39.8	4.87
3	55.948604	9 117 859.7	2.7825	30.8	3.80
4	55.989908	9 006 707.0	3.8232	24.9	3.09
5	56.042961	8 898 363.7	4.8519	20.8	2.58
7	56.184157	8 688 360.4	6.8891	15.6	1.94
10	56.482861	8 387 436.8	9.9203	11.2	1.39
20	58.204426	7 493 381.9	19.9604	5.8	.69
30	60.9492205	6 760 021.0	29.9749	4.0	.44
40	64.562595	6 176 785.1	39.9827	3.1	.30
50	68.874372	5 728 383.8	49.9878	2.6	.21
70	78.943767	5 173 667.5	69.9947	2.1	.09
90	90	$5 \cdot 10^6$	90	2.0	0

TABLE C-35.- $h_f = 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	67.074724	15 148 175.8	-0.5645	84.4	9.85
.1	67.074761	15 134 265.0	-.4400	80.3	9.43
.2	67.074872	15 120 559.2	-.3173	76.6	9.03
.3	67.075056	15 107 039.8	-.1960	73.1	8.66
.4	67.075314	15 093 690.6	-.0762	69.9	8.31
.5	67.075647	15 080 496.9	.0423	67.0	7.99
.7	67.076532	15 054 526.6	.2758	61.7	7.40
1	67.078415	15 016 459.0	.6189	55.0	6.65
2	67.089485	14 895 002.7	1.7197	39.8	4.89
3	67.107930	14 778 853.0	2.7816	30.8	3.81
4	67.133740	14 665 978.5	3.8226	24.9	3.10
5	67.166904	14 555 427.6	4.8514	20.8	2.59
7	67.255229	14 339 570.2	6.8888	15.6	1.94
10	67.442373	14 026 365.3	9.9201	11.2	1.39
20	68.528477	13 064 213.4	19.9603	5.8	.69
30	70.284914	12 231 452.2	29.9748	4.0	.44
40	72.638620	11 533 678.8	39.9827	3.1	.30
50	75.499690	10 971 377.3	49.9878	2.6	.21
70	82.343880	10 238 849.8	69.9947	2.1	.09
90	90	10 ⁷	90	2.0	0

TABLE C-36.- $h_f = 2 \cdot 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	76.003726	25 658 697.8	-0.5676	84.5	9.91
.1	76.003747	25 644 785.4	-.4428	80.4	9.47
.2	76.003813	25 631 074.5	-.3199	76.6	9.07
.3	76.003921	25 617 546.7	-.1985	73.2	8.70
.4	76.004074	25 604 185.6	-.0785	70.0	8.35
.5	76.004269	25 590 976.8	.0401	67.0	8.03
.7	76.004791	25 564 966.0	.2739	61.7	7.44
1	76.005901	25 526 812.4	.6173	55.0	6.68
2	76.012425	25 404 850.0	1.7188	39.8	4.91
3	76.023297	25 287 858.0	2.7809	30.8	3.82
4	76.038512	25 173 805.9	3.8221	24.9	3.10
5	76.058064	25 061 744.0	4.8511	20.8	2.60
7	76.110153	24 841 874.6	6.8886	15.6	1.94
10	76.220592	24 520 225.3	9.9200	11.2	1.40
20	76.863429	23 510 576.9	19.9603	5.8	.69
30	77.909533	22 606 393.7	29.9748	4.0	.44
40	79.322858	21 823 149.7	39.9826	3.1	.30
50	81.056301	21 172 366.0	49.9878	2.6	.21
70	85.255038	20 294 448.8	69.9947	2.1	.09
90	90	$2 \cdot 10^7$	90	2.0	0

TABLE C-37.- $h_f = 5 \cdot 10^7$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	83.50245282	56 079 693.3	-0.5699	84.5	9.95
.1	83.50246276	56 065 779.6	-.4450	80.4	9.51
.2	83.50249258	56 052 064.7	-.3219	76.7	9.11
.3	83.50254227	56 038 530.3	-.2004	73.2	8.73
.4	83.50261184	56 025 160.1	-.0803	70.0	8.38
.5	83.50270129	56 011 939.4	.0384	67.0	8.06
.7	83.50293982	55 985 897.0	.2724	61.7	7.46
1	83.50344669	55 947 676.4	.6161	55.0	6.70
2	83.50642799	55 825 319.7	1.7180	39.8	4.92
3	83.51139577	55 707 670.9	2.7805	30.8	3.83
4	83.51834847	55 592 700.6	3.8218	24.9	3.11
5	83.52728388	55 479 460.2	4.8509	20.8	2.60
7	83.55109090	55 256 459.3	6.8885	15.6	1.95
10	83.60158079	54 928 210.5	9.9200	11.2	1.40
20	83.89584061	53 881 213.1	19.9603	5.8	.69
30	84.37598550	52 920 130.2	29.9748	4.0	.44
40	85.02701023	52 067 614.7	39.9826	3.1	.30
50	85.82871958	51 343 740.5	49.9878	2.6	.21
70	87.78191415	50 342 611.2	69.9947	2.1	.09
90	90	$5 \cdot 10^7$	90	2.0	0

TABLE C-38.- $h_f = 10^8$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
0	86.56175117	106 250 348.0	-0.5708	84.5	9.96
.1	86.56175641	106 236 433.7	-.4459	80.4	9.53
.2	86.56177214	106 222 717.3	-.3227	76.7	9.12
.3	86.56179836	106 209 180.3	-.2012	73.2	8.75
.4	86.56183506	106 195 806.3	-.0810	70.0	8.40
.5	86.56188224	106 182 581.0	.0378	67.1	8.07
.7	86.56200808	106 156 526.1	.2718	61.7	7.47
1	86.56227546	106 118 278.7	.6156	55.0	6.71
2	86.56384817	105 995 765.3	1.7177	39.8	4.93
3	86.56646882	105 877 855.4	2.7803	30.8	3.83
4	86.57013660	105 762 520.0	3.8217	24.9	3.11
5	86.57485038	105 648 811.0	4.8508	20.8	2.60
7	86.58740982	105 424 564.8	6.8884	15.6	1.95
10	86.61404726	105 093 690.1	9.9199	11.2	1.40
20	86.76933006	104 031 801.9	19.9602	5.9	.69
30	87.02283640	103 047 934.4	29.9748	4.0	.44
40	87.36680222	102 167 517.7	39.9826	3.1	.30
50	87.79071500	101 414 010.0	49.9878	2.6	.21
70	88.82467285	100 362 471.6	69.9947	2.2	.09
90	90	10^8	90	2.0	0

TABLE C-39.- $\rho_M = 5 \cdot 10^4$ METERS, $h_i = 2000$ METERS

E_{Mi} , deg	h_f , meters	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
-2	417.44	49 989.00	-2.03854	11.00	0.673
-1.5	854.17	49 989.31	-1.53783	10.69	.660
-1	1290.97	49 989.61	-1.03713	10.39	.648
-.5	1727.80	49 989.90	-.53645	10.10	.636
a-.19	1998.63	49 990.07	-.22603	9.93	.629
0	2164.62	49 990.17	-.03578	9.83	.625
.5	2601.41	49 990.44	.46487	9.56	.613
1	3038.13	49 990.69	.96550	9.31	.602
1.5	3474.74	49 990.94	1.46612	9.06	.591
2	3911.22	49 991.18	1.96673	8.82	.581
3	4783.64	49 991.63	2.96790	8.37	.560
4	5655.11	49 992.05	3.96902	7.95	.541
5	6525.38	49 992.44	4.97009	7.56	.522
7	8261.25	49 993.14	6.97209	6.86	.487
10	10 849.43	49 994.03	9.97477	5.97	.440

a E_m undergoes a sign reversal.

TABLE C-40.- $h_f = 0$ AND $h_i = 10^6$ METERS

E_{Mi} , deg	E_{Mf} , deg	ρ , meters	E , deg	$\rho_M - \rho$, meters	$E_{Mi} - E$, mrad
-30.2	-1.765735	3 548 436.74	-30.21095	42.6	0.191
-30.3	-3.129892	3 397 986.79	-30.30624	29.8	.109
-30.4	-4.061635	3 301 424.40	-30.40454	24.6	.079
-30.5	-4.818691	3 225 995.86	-30.50362	21.4	.063
-30.6	-5.474025	3 162 690.82	-30.60304	19.3	.053
-30.8	-6.597125	3 058 184.24	-30.80232	16.4	.040
-31	-7.560966	2 972 278.08	-31.00189	14.5	.033
-31.5	-9.574616	2 803 354.69	-31.50132	11.7	.023
-32	-11.257673	2 672 474.60	-32.00103	10.1	.018
-32.5	-12.741832	2 564 392.99	-32.50085	8.9	.015
-33	-14.090093	2 471 863.07	-33.00072	8.1	.013
-34	-16.509292	2 318 520.85	-34.00057	7.0	.010
-35	-18.676598	2 193 997.16	-35.00047	6.2	.008
-37	-22.539725	1 998 980.24	-37.00035	5.2	.006
-40	-27.634711	1 786 928.80	-40.00026	4.3	.005
-45	-35.138623	1 548 038.79	-45.00018	3.5	.003
-50	-41.980078	1 387 165.67	-50.00014	3.0	.002
-55	-48.445251	1 271 870.88	-55.00011	2.7	.002
-60	-54.672800	1 186 633.84	-60.00008	2.5	.001
-70	-66.700164	1 075 474.12	-70.00005	2.2	.001
-90	-90	10^6	-90	2.0	0